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Accelerating the Composite Power System Planning by Benders Decomposition

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ABSTRACT
This paper presents an application of Benders decomposition to deal with the complexities in the simultaneous Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP). Unlike the power system operation fields of study, the power system planning methods are not expected to be fast. However, it is always preferable to speed up computations to provide more analysis options for the planner. In this study, Benders decomposition has been applied to solve a mixed integer linear programming formulation of simultaneous GEP & TEP problem. The method has been tested on two test systems: Garver 6-bus system and IEEE 30-bus system. The results are compared to the unified solution of the problem formulation to show the consequent improvements from Benders decomposition.

KEYWORDS: Benders decomposition, Generation expansion planning, Integrated power system planning, Reliability evaluation, Transmission expansion planning.

1. INTRODUCTION

The power system expansion planning has always been an important subject during the history of these systems. Two basic objectives of the planning procedure have been minimization of the expansion costs and maximization of the system reliability. With the emergence of deregulated power systems, the first objective has been correspondingly changed in some systems to the maximization of both profit (by private investors) and the social welfare (by system operator). The current paper does not discuss the expansion planning of restructured power systems. Instead, the method is dedicated to the systems with central planning entity responsible for the capacity addition in both the generation and the transmission sub-systems.

It has been traditional for a long time to unbundle the expansion of power systems into three distinct levels [1]: Generation Expansion Planning (GEP), Transmission Expansion Planning (TEP), and Distribution Expansion Planning (DEP). Two main reasons for this separation are [1,2]:

1. GEP requires a much more amount of financial resources in comparison with TEP.
2. The integrated planning is too complicated to be solved in a unified mathematical context.

Since DEP involves a lot of geographical and technical complexities, it seems logical to segregate it from the previous levels, at the moment. Nevertheless, in [3] the effect of micro-grid expansion in distribution systems has been considered besides the GEP and TEP problems. Because of the computer hardware & software progressions during the recent years, some theoretical advancements have been also done in the simultaneous GEP/TEP problem. In [2] a static Mixed Integer Linear Programming (MILP) model has been proposed to first minimize the expansion costs and then enhance the reliability of the economically optimal plan. The method is very efficient in the modeling of the planner's requirements but it is very slow in the convergence. The proposed model in the current work has been established based on the model in [2] to provide a benchmark for the comparison.

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One of the early MILP frameworks for the simultaneous GEP/TEP problem has been proposed in [4]. The method is very remarkable with regard to the available facilities in the time of publication. However, it neglects the adequacy requirements of the planning phase. In [5] one of the first applications of the Benders decomposition to this problem has been shown. The problem has been decomposed into two sub-problems. The first sub-problem specifies the optimal capacity addition (with the least investment cost), and the second sub-problem evaluates the optimal operation schedule (with the least operation cost). The shortcoming of the previous research work (i.e., neglecting the adequacy assessment) also exists in this paper. This deficiency has been overcome in [6] which is one of the best models with regard to the reliability considerations at Hierarchical Level II (HLII). Similar to [2], the computation burden of this MILP formulation is also significant. The application of the expert systems has been discussed in [7,8]. These techniques will always lead to an expansion plan but it is not guaranteed that the plan is always the optimal one. That is why expert-based expansion planning has been abolished. The simultaneous GEP/TEP has been also a popular discussion in the market environment. Since this paper is not designed for this category of models, the reader is encouraged to see [9-13] for more details. Another distinguished idea has been presented in [14] with a nonlinear model which can compute fuel transportation costs based on the consumption locations. Besides the model, the solution algorithm is also innovative. However, the N-1 contingency analysis has been employed to check system security. Hence, no quantitative reliability measure has been used at HLII. In [15], a multi-objective model has been introduced to solve a single-objective problem. The constraint violations have been added together to form the second objective function and it is minimized besides the main objective function. It is an interesting idea, but it is not clear why a problem formulation which can be solved in a single objective framework should be solved with multi-objective optimization techniques. Transmission Switching (TS) has been modeled in the simultaneous GEP/TEP problem in [16]. Similar to the previous works of this research team, the solution method is Benders decomposition. The master-problem includes the candidate elements to be invested in generation and transmission systems. The sub-problem includes the system security check, and also evaluation of the optimal dispatch. TS has been considered in both the master-problem and the sub-problem. The model is mathematically attractive but it ignores any reliability measure. This weakness has been overcome in [17] with generating a reliability cut in the sub-problem and adding it to the subsequent iteration of the master-problem. However, this reliability assessment is restricted to HLI (i.e., reliability of the generation system). On the other hand, the presented work in [3] is a more advanced model by the same research team which has been discussed earlier. The application of the Superconducting Fault Current Limiter (SFCL) is another example in which Benders decomposition has been used to solve a simultaneous GEP/TEP problem [18]. Here, the master-problem minimizes the expansion costs while the sub-problem limits the fault current by modeling the effect of SFCL on the system impedance matrix. Based on the SFCL capability in controlling the fault current, a cut will be added to the next iteration of the master-problem. The authors have implied that technical, and economical issues have prevented the application of SFCLs in reality. In addition to the applicability of the model, it does not include any reliability evaluation for the planning study.

This paper aims to speed up the expansion planning computations in the presence of the reliability assessment at HLII. To achieve this goal, Benders decomposition has been utilized as a powerful tool in solving MILP problems. The remaining of this paper has been organized as follows: Section 2 introduces the problem formulation as a simultaneous GEP/TEP model. In Sec. 3, two solution methodologies has been discussed: first a unified MILP solution method has been evaluated and then the basic model has been partitioned into appropriate parts to be solved by Benders decomposition. Section 4 provides numerical results from the presented methods and compares them with the each other. Finally, Sec. 5

concludes this research work.

2. PROBLEM FORMULATION

The expansion planning model used in this paper is similar to the presented model in [2]. The reader is encouraged to refer to this article for a detailed explanation of the model.

The objective function of the model is formed by the sum of investment costs, (1) and operation costs, (2) for the generating units plus the investment cost of the newly added transmission lines (3):

$$C_{i,gtech,ng}^{Investment} = \sum_{y=1}^{Y} \Phi_{gtech,y}^{feasible} \cdot \zeta_{i,gtech,ng,y}^{G} \quad (1)$$

$$C_{i,gtech,ng}^{Operation} = \alpha_{gtech} \cdot \Lambda_{i,gtech,ng} \quad (2)$$

$$C_{i,j,tc,type,nc}^{Investment} = l_{i,j} \cdot \Phi_{tc,type}^{feasible} \cdot \zeta_{i,j,tc,type,nc}^{T} \quad (3)$$

where, *gtech* and *y* indices denoting available generation technologies and capacities, respectively. Also, *ng* and *nc* are indices for counting number of added units and transmission lines and *i, j* indices to show system buses. Finally, *tc* and *type* are indices denoting available transmission capacities, and reactances. $\left\{\Phi_{gtech,y}^{feasible}\right\}$ and $\left\{\Phi_{tc,type}^{feasible}\right\}$ are the cost sets for the feasible generation and transmission facilities, respectively. α_{gtech} is cost of producing 1 MWh energy by technology *gtech*. $\Lambda_{i,gtech,ng}$ is the amount of produced energy by the ng_{th} unit of technology *gtech* on bus *i*. $l_{i,j}$ is the length of the transmission line between buses *i* and *j*. $\zeta_{i,gtech,ng,y}^{G}$ and $\zeta_{i,j,tc,type,nc}^{T}$ are binary decision variables in generation and transmission systems, respectively. If $\zeta_{i,gtech,ng,y}^{G}$=1, the ng_{th} unit of technology *gtech* on bus *i* is constructed from capacity *y*. This generation capacity is later denoted by $\Delta P_{i,gtech,ng}^{G}$. If $\zeta_{i,j,tc,type,nc}^{T}$=1, the nc_{th} circuit between buses *i* and *j* is constructed from capacity *tc* and reactance *type*. This transmission capacity and sceptance are later denoted by $\Delta P_{i,j,nc}^{T}$, and $\Delta B_{i,j,nc}$. The model constraints are discussed below:

$$\Delta P_{i,gtech,ng}^{G} = \sum_{y=1}^{Y} P_{gtech,y}^{feasible} \cdot \zeta_{i,gtech,ng,y}^{G} \quad (4)$$

$$\Delta P_{i,j,nc}^{T} = \sum_{tc=1}^{\left\|P_{tc}^{feasible}\right\|} \sum_{type=1}^{\left\|S_{tc,type}^{feasible}\right\|} P_{tc}^{feasible} \cdot \zeta_{i,j,tc,type,nc}^{T} \quad (5)$$

$$\Delta B_{i,j,nc} = \sum_{tc=1}^{\left\|P_{tc}^{feasible}\right\|} \sum_{type=1}^{\left\|S_{tc,type}^{feasible}\right\|} S_{tc,type}^{feasible} \cdot \zeta_{i,j,tc,type,nc}^{T} \quad (6)$$

where, $\left\{P_{tc}^{feasible}\right\}$, $\left\{S_{tc,type}^{feasible}\right\}$ are sets of feasible transmission capacities and reactances, respectively. Also, $\left\{P_{gtech,y}^{feasible}\right\}$ is the set of feasible generation capacities.

$$\sum_{y=1}^{Y} \zeta_{i,gtech,ng,y}^{G} \le 1 \quad (7)$$

$$\sum_{tc=1}^{\left\|P_{tc}^{feasible}\right\|} \sum_{type=1}^{\left\|S_{tc,type}^{feasible}\right\|} \zeta_{i,j,tc,type,nc}^{T} \le 1 \quad (8)$$

The acquirable energy from the ng_{th} unit of technology *gtech* on bus *i* ($\Lambda_{i,gtech,ng}$) should be placed in a specific range:

$$\eta_{gtech} \cdot MOH_{gtech} \cdot \Delta P_{i,gtech,ng}^{G} \le \Lambda_{i,gtech,ng} \quad (9)$$

$$\Lambda_{i,gtech,ng} \le MOH_{gtech} \cdot \Delta P_{i,gtech,ng}^{G} \quad (10)$$

η_{gtech} is the contribution factor (capacity factor) of generation technology *gtech* in energy production. MOH_{gtech} is the maximum operation hours of generation technology *gtech* in the planning horizon. The total producible energy should be greater than the Total Required Energy (TRE):

$$\sum_{i=1}^{n} \sum_{gtech \in \left\{P_{gtech,y}^{feasible}\right\}} \sum_{ng=1}^{NG} \Lambda_{i,gtech,ng} \ge TRE \quad (11)$$

The fuel consumption in the thermal units should be kept less than the available fuel ($Fl^{\max}$) in the planning horizon:

$$\sum_{i=1}^{n} \sum_{gtech \in \left\{P_{gtech,y}^{feasible}\right\}} \sum_{ng=1}^{NG} \psi_{gtech} \cdot \Lambda_{i,gtech,ng} \le Fl^{\max} \quad (12)$$

ψ_{gtech} is the fuel consumption rate by generation technology *gtech* for 1 MWh energy production. Nowadays, one of the most important issues in the power system planning is to control the emission of pollutants from the thermal units:

$$\sum_{i=1}^{n} \sum_{gtech \in \left\{P_{gtech,y}^{feasible}\right\}} \sum_{ng=1}^{NG} \rho_{gtech}^{SO_2} \cdot \Lambda_{i,gtech,ng} \le \delta_{SO_2}^{\max} \quad (13)$$

$$\sum_{i=1}^{n} \sum_{gtech \in \left\{P_{gtech,y}^{feasible}\right\}} \sum_{ng=1}^{NG} \rho_{gtech}^{NO_X} \cdot \Lambda_{i,gtech,ng} \le \delta_{NO_X}^{\max} \quad (14)$$

In (13) and (14), $\rho_{gtech}^{SO_2}$ and $\rho_{gtech}^{NO_X}$ are the amount of produced SO_2 and NO_X by generation technology *gtech* for 1 MWh energy production. In this paper, SO_2 and NO_X have been considered as the targeted pollutants to be controlled but obviously any other pollutant may be added to the model, in the same way. $\delta_{SO_2}^{\max}$ and $\delta_{NO_X}^{\max}$ are the maximum allowable amount of these pollutants to be emitted in the atmosphere. Evidently, a unit's power generation is limited to its capacity:

$$0 \le P_{i,gtech}^{G} \le \sum_{ng=1}^{NG} \Delta P_{i,gtech,ng}^{G} \quad (15)$$

Transmission network flows should be also constrained to the line's thermal limits. For the existing transmission lines (16) and for the newly constructed transmission lines (17), imply this constraint.

$$-P_{i,j,nc}^{\max} \le B_{i,j,nc} \cdot \left(\theta_i - \theta_j\right) \le P_{i,j,nc}^{\max} \quad (16)$$

$$-\Delta P_{i,j,nc}^{T} \le \Delta B_{i,j,nc} \cdot \left(\theta_i - \theta_j\right) \le \Delta P_{i,j,nc}^{T} \quad (17)$$

In (16), $P_{i,j,nc}^{\max}$ shows the existing capacity of the nc_{th} circuit between buses *i* and *j*. Finally, the DC power flow constraint is given by (18):

$$\left[B + \Delta B\right]_{n\times n} \cdot [\theta]_{n\times 1} \quad \left[P_G + \Delta P_G - P_D - \Delta P_D\right]_{n\times 1} \quad (18)$$

Here, ΔP_D is the forecasted load growth over the planning horizon. The whole problem formulation is represented by:

$$\begin{aligned} Minimize \; & \sum_{i=1}^{n} \sum_{gtech \in \{P_{gtech,y}^{feasible}\}} \sum_{ng=1}^{NG} \left(C_{i,gtech,ng}^{Investment} + C_{i,gtech,ng}^{Operation}\right) + \\ & \sum_{i=1}^{n} \sum_{j \; i+1}^{n} \sum_{tc \in \{P_{tc}^{feasible}\}} \sum_{type \in \{S_{tc,type}^{feasible}\}} \sum_{nc=1}^{NC} C_{i,j,tc,type,nc}^{Investment} \quad (19) \\ subject \; to: & \\ & (1)-(18) \end{aligned}$$

The decision variables are $\zeta_{i,gtech,ng,y}^{G}$, $\zeta_{i,j,tc,type,nc}^{T}$, $\Lambda_{i,gtech,ng}$, and θ_i. The first two groups reveal generation expansion and transmission expansion, respectively. It is possible to define some operational constraints on θ_i. However, these voltage phase angels have no detailed significance in the planning process. As the final point in this Sec. it should be noted that the reliability assessment algorithm is not rewritten in the current article for the sake of briefness. The reader is encouraged to study this algorithm in [2].

3. SOLUTION METHODOLOGIES

In this Sec., two solution methods have been presented for the model. First a united solution method solves the planning model in a MILP framework by means of some auxiliary variables and equations. Then, Benders decomposition has been used to partition the original problem formulation, (19), to a MILP master-problem and a LP sub-problem.

3.1. Unified solution method

The problem formulation, (19), includes two sets of nonlinear constraints: (17) and (18). In the both sets, a discrete variable, $\Delta B_{i,j,nc}$, is multiplied by a continuous variable, θ. Since the transmission facilities are available in finite number of options (i.e., discrete capacities in $\left\{P_{tc}^{feasible}\right\}$, and discrete per length unit reactance in $\left\{S_{tc,type}^{feasible}\right\}$), it is possible to eliminate the nonlinearity in the both constraints. To achieve this goal, an auxiliary variable, $Z_{i,j,nc} = \Delta B_{i,j,nc} \cdot \Delta\theta_j$, should be defined with the following relations:

$$Z_{i,j,nc} + M \cdot \zeta_{i,j,tc,type,nc}^{T} \le M + S_{tc,type}^{feasible} \cdot \Delta\theta_j \quad (20)$$

$$Z_{i,j,nc} - M \cdot \zeta_{i,j,tc,type,nc}^{T} \ge -M + S_{tc,type}^{feasible} \cdot \Delta\theta_j \quad (21)$$

$$Z_{i,j,nc} - M \cdot \sum_{tc=1}^{\left\|P_{tc}^{feasible}\right\|} \sum_{type=1}^{\left\|S_{tc,type}^{feasible}\right\|} \zeta_{i,j,tc,type,nc}^{T} \le 0 \quad (22)$$

$$Z_{i,j,nc} + M \cdot \sum_{tc=1}^{\left\|P_{tc}^{feasible}\right\|} \sum_{type=1}^{\left\|S_{tc,type}^{feasible}\right\|} \zeta_{i,j,tc,type,nc}^{T} \ge 0 \quad (23)$$

In the above equations, M denotes a large enough number. It is sometimes called the big M in the literature. According to the definition of $Z_{i,j,nc}$, the following relation should be also held:

$$Z_{i,i} = \sum_{\substack{j=1 \\ j\ne i}}^{n} \sum_{nc=1}^{NC} Z_{i,j,nc} \quad (24)$$

Now, this auxiliary variable can be employed to remove the nonlinearity in (17):

$$-\Delta P_{i,j,nc}^{T} \le Z_{j,i,nc} - Z_{i,j,nc} \le \Delta P_{i,j,nc}^{T} \quad (25)$$

Similarly in (18), wherever $\Delta B_{i,j,nc}$ is multiplied by θ_j, the variable $Z_{i,j,nc}$ is used instead. As an instance, the active power balance on bus i can be written as:

$$\sum_{j=1}^{n}\sum_{nc=1}^{NC}\left[B_{i,j,nc}\cdot\left(\theta_i-\theta_j\right)+Z_{j,i,nc}-Z_{i,j,nc}\right]= \\ P_{G,i}+\sum_{gtech\in\left\{P_{gtech,y}^{feasible}\right\}}\sum_{ng=1}^{NG}\Delta P_{i,gtech,ng}^{G}-P_{D,i}-\Delta P_{D,i} \tag{26}$$

By replacing (17) and (18) in (25), and (26) in (19) and including (20)-(24) the MILP problem formulation is obtained as follows:

$$\begin{aligned} Minimize\ & \sum_{i=1}^{n}\sum_{gtech\in\{P_{gtech,y}^{feasible}\}}\sum_{ng=1}^{NG}\left(C_{i,gtech,ng}^{Investment}+C_{i,gtech,ng}^{Operation}\right)+ \\ & \sum_{i=1}^{n}\sum_{j\ \ i+1}^{n}\sum_{tc\in\{P_{tc}^{feasible}\}}\sum_{type\in\{S_{tc,type}^{feasible}\}}\sum_{nc=1}^{NC}C_{i,j,tc,type,nc}^{Investment} \\ subject\ to: & \\ & (1)-(16) \\ & (20)-(26) \end{aligned} \tag{27}$$

3.2. Application of Benders decomposition

Benders decomposition is an efficient decomposition method which was firstly proposed by Benders [19] in 1962 for mixed integer optimization problems. In its general form (called generalized Benders decomposition proposed by Geoffrion [20] in 1972) it can be used to solve mixed integer nonlinear programming (MINLP) problems with a special structure. The problem is decomposed into a pure integer master-problem (MILP part) and a NLP sub-problem. The complete methodology can be reviewed in [21] and has been widely used in the engineering applications. In the power system literature, this decomposition approach has been also used very often [22]-[27]. In this subsection, the elimination of the nonlinearities in (19), by means of Benders decomposition, has been described.

Since $\Delta B_{i,j,nc}$ is related to the binary variable $\zeta_{i,j,tc,type,nc}^{T}$, as shown in (6), there will be no nonlinear term in (17) & (18) if the value of this variable has been assessed. Consider the problem (19) in the following structure:

$$\begin{aligned} & minimize\ \ F(x,y)=c^T\cdot x+d^T\cdot y \\ & subject\ to: \\ & \quad h(x,y)=0 \\ & \quad g(x,y)\le 0 \\ & \quad x\ge 0,\ y\in\{0,1\} \end{aligned} \tag{28}$$

Note that x and y are the set of continuous (i.e., $\Lambda_{i,gtech,ng}$) and binary (i.e., $\zeta_{i,gtech,ng,y}^{G}$ and $\zeta_{i,j,tc,type,nc}^{T}$) variables, respectively. h is the set of equality constraints (i.e., active power balance, (18)) and g is the set of inequality constraints. In the first iteration, the binary variables are set randomly ($y=y^{(0)}$) and the lower bound for the objective function is set to $d^T\cdot y^{(0)}$. Then the LP sub-problem is formed with θ, and $\Lambda_{i,gtech,ng}$ as the set of (continuous) variables:

$$\begin{aligned} & minimize\ \ c^T\cdot x \\ & subject\ to: \\ & \quad h(x,y^{(0)})=0 \\ & \quad g(x,y^{(0)})\le 0 \\ & \quad x\ge 0 \end{aligned} \tag{29}$$

There are two possibilities for (29): it is feasible, or it is infeasible. If the sub-problem is feasible, an optimality cut will be generated by using the set of Lagrangian multipliers μ, and λ for equality, and inequality constraints, respectively:

$$d^T\cdot y\ge \\ F(x^{(0)},y^{(0)})+\mu^T\cdot h(x^{(0)},y)+\lambda^T\cdot g(x^{(0)},y) \tag{30}$$

The upper bound of the objective function is set to $c^T\cdot x^{(0)}+d^T\cdot y^{(0)}$ in this case. If the sub-problem is infeasible, the following LP problem should be solved to minimize the weighted sum of infeasibilities:

$$\begin{aligned} & minimize\ \ \sum_k w_k\cdot\beta_k \\ & subject\ to: \\ & \quad h(x,y^{(0)})=0 \\ & \quad g_k(x,y^{(0)})\le\beta_k \\ & \quad \sum_k w_k=1 \\ & \quad x,\beta_k\ge 0 \end{aligned} \tag{31}$$

where, w_k and β_k as the weighting factor and the infeasibility of the k_{th} inequality constraint, respectively. From (31) and similar to (30), a feasibility cut will be generated:

$$0\ge\mu^T\cdot h(x^{(0)},y)+\lambda^T\cdot g(x^{(0)},y) \tag{32}$$

The optimality cut (30), or the feasibility cut (32), will be added to the next iteration of the MILP master-problem with $\zeta_{i,gtech,ng,y}^{G}$ and $\zeta_{i,j,tc,type,nc}^{T}$ as the set of (binary) variables:

$$
\begin{aligned}
& \textit{minimize} \;\; d^T \cdot y \\
& \textit{subject to}: \\
& \quad h(y) = 0 \\
& \quad g(y) \le 0 \\
& \quad d^T \cdot y \ge \\
& \quad F(x^{(k)}, y^{(k)}) + \mu^T \cdot h(x^{(k)}, y) + \lambda^T \cdot g(x^{(k)}, y), \\
& \qquad\qquad\qquad\qquad k = 1, \ldots, K \\
& \quad 0 \ge \mu^T \cdot h(x^{(l)}, y) + \lambda^T \cdot g(x^{(l)}, y), \\
& \qquad\qquad\qquad\qquad l = 1, \ldots, L \\
& \quad y \in \{0,1\}
\end{aligned} \tag{33}
$$

The k and l indices are dedicated to the optimality and feasibility cuts of the previous iterations, respectively. The solution from (33) is used to solve (29) in the next iteration and this iterative procedure is continued till the difference between the lower and upper bounds of the objective function becomes negligible. For the sake of guidance, the master-problem in the first iteration is:

$$
\begin{aligned}
& \textit{Minimize} \sum_{i=1}^{n} \sum_{gtech \in \{P_{gtech,y}^{feasible}\}} \sum_{ng=1}^{NG} C_{i,gtech,ng}^{Investment} + \\
& \qquad \sum_{i=1}^{n} \sum_{j \; i+1}^{n} \sum_{tc \in \{P_{tc}^{feasible}\}} \sum_{type \in \{S_{tc,type}^{feasible}\}} \sum_{nc=1}^{NC} C_{i,j,tc,type,nc}^{Investment} \\
& \textit{subject to}: \\
& \qquad (1),(3),(7),(8)
\end{aligned} \tag{34}
$$

The optimal solution to (34) is given by $\zeta_{i,gtech,ng,y}^{G(*)}$ and $\zeta_{i,j,tc,type,nc}^{T(*)}$. These values are then replaced in (4)-(6) to evaluate $\Delta P_{i,gtech,ng}^{G}$, $\Delta P_{i,j,nc}^{T}$, and $\Delta B_{i,j,nc}$. After this evaluation, the sub-problem is formed:

$$
\begin{aligned}
& \textit{Minimize} \sum_{i=1}^{n} \sum_{gtech \in \{P_{gtech,y}^{feasible}\}} \sum_{ng=1}^{NG} C_{i,gtech,ng}^{Operation} \\
& \textit{subject to}: \\
& \qquad (2),(9)-(18)
\end{aligned} \tag{35}
$$

The cuts is generated based on the feasibility status of (35), as described previously and the procedure is continued until converging to the optimal solution. The convergence proof for this structure can be found in the optimization textbooks [21].

4. SIMULATION RESULTS

This section presents the simulation results to provide a comparison between the two introduced solution methods in Sec. 3. The benchmarks are a small 6-bus test system (known as the Garver test system in the TEP literature) and IEEE 30-bus test system. The planning horizon in the both cases is assumed to be 15 years. The available generation and transmission technologies and their costs have been adopted from [2]. The proposed Benders decomposition algorithm besides the unified model have been both implemented in GAMS environment and they are solved by means of CPLEX solver.

4.1. Garver test system

Figure 1 shows this test system before the expansion planning. The system peak load is considered to be 800 MW. The only permitted location for the construction of the hydro power plants is bus #1. The hydro capacity has been limited to 200 MW. At the beginning of the planning period, there exist 3×30 MW gas turbine and 1×60 MW steam turbine units on bus #1. Also, 2×60 MW steam turbine units exist on bus #3.

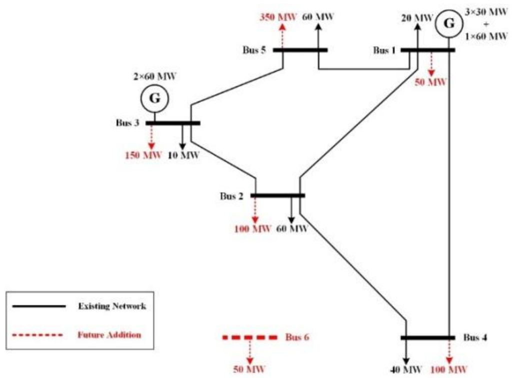

Fig. 1. 6-bus test system.

Since this is a small system, the united solution methodology explained in 3.1 subsection, and given by (27), can effectively handle the problem complexities. Table 1 shows the optimal plan for this test system. The generation technologies have been abbreviated as: hydro (H), gas turbine (G), steam turbine (S), combined cycle (CC) and nuclear (N).

The same results, as presented in Table 1, has been evaluated by employing benders decomp-osition approach. For a system with Intel core i7 CPUs, and 6 GB of RAM, the simulation runtime is 32 minutes for the unified method and 55 minutes for the Benders decomposition. The convergence path has been depicted in Fig. 2. In this case, the problem is small enough to be solved with the united optimization. Hence, employing Benders decomposition will not provide a mentionable advantage.

Table 1. The optimal expansion plan for 6-bus test system.

Bus No.	Technology	Capacity (MW)	Cost (million $)
1	H	1×60+2×70	362.1
3	S	2×150	488.5
5	CC	2×350+1×400	1534.7
From-to	Reactance (p.u.)	Capacity (MW)	Cost (million $)
4-6	j0.312	80	5.6
2-5	j0.252	120	7.2

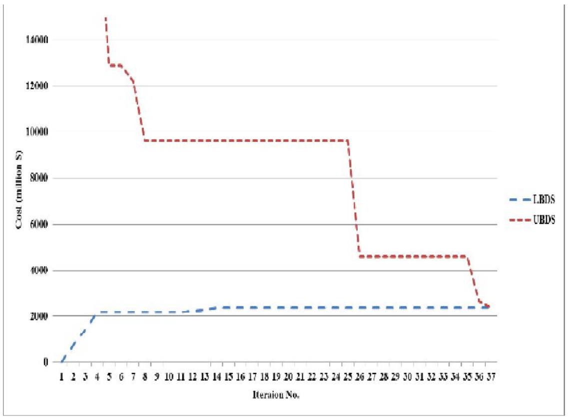

Fig. 2. The convergence path for the Benders decomposition, 6-bus test system (LBDS: lower bounds, UBDS: upper bounds).

4.2. IEEE 30-bus test system

Figure 3 depicts this test system before the expansion planning. The system peak load is considered to be 2800 MW. The permitted locations for the construction of the hydro power plants are buses #13, #14, and #15. The hydro capacity has been limited to 600 MW.

Table 2 demonstrates the optimal solution for this test system. The same as the previous case study and because of the linear formulation, Benders decomposition leads to the exactly the same optimal plan. The unified optimization requires more than 10 hours to find the best solution. However, Benders decomposition converges to the optimal plan in about 4 hours and 35 minutes. Figure 4 shows the convergence path for the IEEE 30-bus test system. This case study reveals the importance of Benders decomposition in solving the complex planning models.

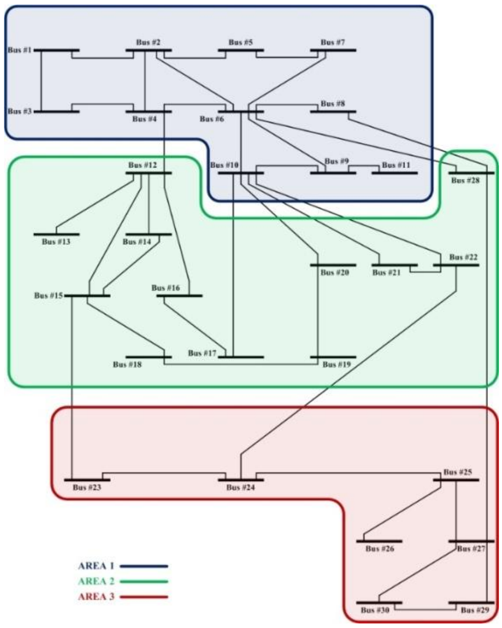

Fig. 3. IEEE 30-bus test system.

Table 2. The optimal expansion plan for IEEE 30-bus test system.

Bus No.	Technology	Capacity (MW)	Cost (million $)
1	S	1×125	203.5
4	G	2×50	84.3
6	CC	2×350	976.6
6	G	2×50+1×30	62.1
8	G	1×60	50.6
9	G	2×60	78.6
10	CC	1×250	348.8
13	H	3×60+4×70	832.8
14	H	2×50+2×70	434.5
15	H	2×50	181.1
18	CC	1×250	348.8
20	G	2×50	84.3
21	S	1×200	325.7
23	CC	1×160	223.2
26	N	1×1000	1345.8
26	G	1×30	24.2
28	G	2×50	63.8
29	G	3×30	59.1
From-to	Reactance (p.u.)	Capacity (MW)	Cost (million $)
2-9	j0.136	100	5.2
4-8	j0.179	150	8.6
10-16	j0.068	80	4.8
13-19	j0.084	120	6.3
18-26	j0.115	120	6.6
19-26	j0.234	150	9.2
23-30	j0.307	180	10.4

When considering the reliability computations at

HLII the model becomes extensive even for the small systems. Therefore, Benders decomposition is an efficient solution methodology in highly complicated practical planning models.

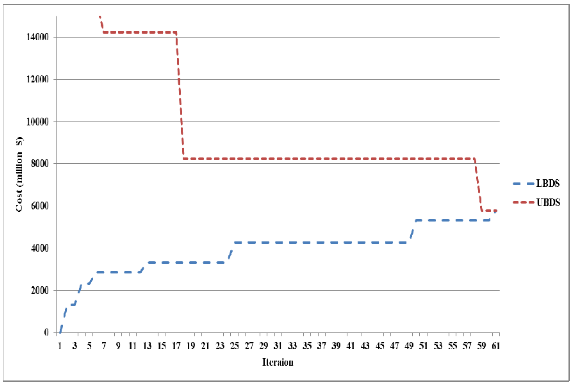

Fig. 4. The convergence path for the Benders decomposition, 30-bus test system (LBDS: lower bounds, UBDS: upper bounds).

5. CONCLUSIONS

This paper reflects the efficiency of the Benders decomposition technique as a useful mathematical method in the planning studies. The method saves a considerable amount of time and energy especially in the large case studies. The importance of the presented decomposition technique becomes more clear by taking the reliability assessment of the composite system into consideration. It has been demonstrated that when a MILP framework should be solved iteratively (as in the reliability reinforcement algorithm), the decomposition technique becomes even more efficient. Although, the planning studies are considered to be offline, holding fast enough optimization methods help the planner to implement and analyze more details in his/her modeling survey. Further studies can be concentrated on performing such a comparison for the market based planning methods.

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