

## An Advanced State Estimation Method Using Virtual Meters

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**Abstract-** Power system state estimation is a central component in energy management systems of power system. The goal of state estimation is to determine the system status and power flow of transmission lines. This paper presents an advanced state estimation algorithm based on weighted least square (WLS) criteria by introducing virtual meters. For each bus of network, except slack bus, a virtual meter is considered, using the concept of KCL law. Regarding virtual meter, an improved state estimation algorithm is obtained with higher accuracy and lower computation burden. In the case study, at first, a simple 6-bus test system is presented and the proposed state estimation algorithm is followed step by step. Then, in order to evaluate the effectiveness and applicability of algorithm, IEEE 30-bus and IEEE 118-bus test systems are also taken into consideration. The obtained results verify the usefulness of the proposed method in large size power systems including thousands of buses.

**Keyword:** Measurement error, Meter measurement, State estimation, Virtual meter, Voltage phase angle.

### 1. INTRODUCTION

This document provides an example of the desired layout for a JOAPE. The definition of state estimation is assignment of a value to an unknown variable based on measurements data taken from that system. A state estimator receives field measurement data from remote terminal units through data transmission systems, such as a supervisory control and data acquisition (SCADA) system [1]. Generally, the solution methodology of state estimation are based on two groups namely, mathematical methods and intelligent methods. Weighted Least Squares (WLS), Weighted least Absolute Value (WLAV) and Estimation with Non-Fixed Error (M-Estimator) belong to first group [1-4]. However, the intelligent based works are Neural Network based state estimation [6], Fuzzy Inference System (FIS) [7], and Adaptive Neuron Fuzzy Inference System (ANFIS) [8] and also evolutionary algorithm such as particle swarm optimization [9]. Intelligent based methods state estimation are faster than the mathematical ones but with lower accuracy.

Owing to the important role of state estimation in power systems, so many works are published to study state estimation from different aspects. The real-time

system state evaluation and control for secure operation of sustainable power systems is proposed in [10]. Different methods of state estimations and solutions algorithms are taken in [11,12]. In [13], the phasor measurement unit (PMU) placement in electric power networks is presented in the form of multi-objective model. In [14], the optimal placement of PMU is presented based on branch placement. Also the communication feasibility analysis for smart grid with phasor measurement units is presented in [15]. Optimal location of phasor measurement units (PMUs) is solved in the form of linear programming (LP) problem to decrease the number of measurements while increasing the accuracy of estimation [16]. An autonomous state estimation is used to extract the real-time model of smart in which raw data is filtered to reach a reliable mathematical model [17]. Also the impact of load modeling in distribution state estimation is studied in [18]. A joint estimation of state and parameter with synchrophasor is proposed [19]. Finally, condition monitoring techniques of power transformers is reviewed in [20], indicating the important role of state estimation and condition monitoring of system and elements in power system.

Among mathematical based methods, the Weighted least squares (WLS) state estimation approach has been an important tool for determining the optimal estimate of power system states [21]. State estimation for electric transmission grids was first formulated as a weighted least-squares problem by Fred Schweppe and his research group in 1969 [22]. The Impacts of load levels and topology errors on WLS state estimation

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convergence is assessed in [23]. A new formulation for power system state estimation is proposed based on the regularized least squares method, which can deal with ill-posed problems; eliminating, the mathematical unfeasibility caused by lack of measurements [24]. In [25], an approach has been proposed called extended least squares for estimating parameters of pseudo-linear models which was firstly applied to power system state estimation in [26] to consider measurement errors and model errors.

In the above mentioned papers, and also the other papers in the area, the state estimation issue is considered from different viewpoint. Considering the computation burden decrease and also increasing accuracy of state estimation, and regarding the widely using WLS method in state estimation, in this paper, a new method is proposed, based on WLS for state estimation. The novelty of the proposed method in this paper is its applicability for large size power systems with lower computation burden and more accuracy in comparison with the method presented in [1].

The remainder of this paper is organized as follows. In section 2, maximum likelihood WLS state estimation presented in [1] is reviewed in brief. The proposed method is taken in section 3. The effectiveness of proposed method is studied in small, medium and large size power systems in section 4. Finally, section 5 includes conclusions.

## 2. MAXIMUM LIKELIHOOD WEIGHTED STATE ESTIMATION

In order to present the proposed method, the method for state estimation presented in [1] is first review in brief as follows.

In the maximum likelihood weighted least square state estimation, the objective function is:

$$\min_X J(X) = \sum_{i=1}^{N_m} \frac{[Z_i^{meas} - f_i(X)]^2}{\sigma_i^2} \quad (1)$$

where  $N_{mis}$  the number of measurements,  $X = [x_1, x_2, x_3, \dots, x_{N_s}]$  is state vector and  $x_1, x_2, x_3, \dots, x_{N_s}$  are state variables being estimated. Also  $f_i(X)$  is the function used for calculating the value that is measured by the  $i$ th meter. If  $f(x_1, x_2, x_3, \dots, x_{N_s})$  is a linear function, then it can be written as [1]:

$$f_i(x_1, x_2, x_3, \dots, x_{N_s}) = f_i(X) = h_{i1}x_1 + h_{i2}x_2 + \dots + h_{iN_s}x_{N_s} \quad (2)$$

Accordingly, vector  $f(X)$  can be written as follows.

$$f(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_{N_m}(X) \end{bmatrix} = [H]X \quad (3)$$

where  $[H]$  is matrix  $N_m \times N_s$  including coefficients of linear function  $f(X)$ .

$N_m$  number of measurements

$N_s$  number of variables being estimated

On the other hand:

$$[H]X = Z^{meas} \quad ; \quad Z^{meas} = \begin{bmatrix} Z_1^{meas} \\ Z_2^{meas} \\ \vdots \\ Z_{N_m}^{meas} \end{bmatrix} \quad (4)$$

where  $Z^{meas}$  is measurement matrix  $N_m \times 1$ .

Including the measurement value of the meters in the system, Eq. (1) can be written as [1]:

$$\min_X J(X) = [Z^{meas} - f(X)]^T [R^{-1}] [Z^{meas} - f(X)] \quad (5)$$

where  $[R]$ , is variance matrix indicating the measurement errors of meters [1].

$$[R] = \begin{bmatrix} \sigma_1^2 & & \\ & \sigma_2^2 & \\ & & \ddots \\ & & & \sigma_{N_m}^2 \end{bmatrix} \quad (6)$$

Finally, the estimated value of unknown variables in case of overdetermined, i.e.,  $N_m > N_s$ , can be obtained according to the Eq. (7) [1]:

$$X_{N_s \times 1}^{est} = \left[ [H_{N_m \times N_s}]^T [R_{N_m \times N_m}^{-1}] [H] \right]^{-1} [H]^T [R^{-1}] Z_{N_m \times 1}^{meas} \quad (7)$$

The method in [1] is based on H matrix. Therefore, in the remainder of this paper, the proposed method in [1] is known as H method and the proposed method will be known as H\* method.

## 3. THE PROPOSED METHOD

Using DC load flow model, the active power balance equation in the  $i$ th node of network is:

$$P_{i,Gen} - P_{i,Load} = \sum_{\substack{j=1 \\ j \neq i}}^N f_{ij} \quad i = 1, \dots, N \quad (8a)$$

$$f_{ij} = \frac{1}{x_{ij}} (\theta_i - \theta_j) \quad i = 1, \dots, N \quad (8b)$$

where,  $P_{i,Gen}, P_{i,Load}$  are active power generation and load of the  $i^{th}$  bus, respectively.  $f_{ij}$  is the active power flow of line connecting bus  $i$  to bus  $j$ , and  $x_{ij}$  is the reactance of that line.  $\theta_i, \theta_j$  are voltage phase angle of bus  $i$  and  $j$ , respectively in radian. Considering the right hand side of Eq. 8(a), a virtual meter is introduced at each bus. The measurement of the virtual meter is equal to the sum of the measurements of the meters that measure the flow of lines connected to the interested bus. This matter is better illustrated in Fig 1. In this figure, three lines are connected to the bus whose active flows are measured by meter1, meter 2 and meter 3. Therefore, the measurement of virtual meter for this bus is equal to the sum of measurements of meter1, meter 2 and meter 3. Suppose  $K_i$  is a set of far end nodes of lines connected to the  $i^{th}$  bus of the network and also,  $M_{ik}$  is the measurement of meter installed at line between node  $i$  and  $k$ . Then  $M_i^{VirMeas}$  is defined as the measurements of virtual meter at bus  $i$ , calculated as follows.

$$M_i^{VirMeas} = \sum_{k \in K_i} M_{ik} \quad (9)$$

It is noted that, in order to measure the flow of each line, only one meter is needed, because in the case of using DC power flow, the system is lossless and the active power flow in near end and far end of line is equal in value but with opposite direction.

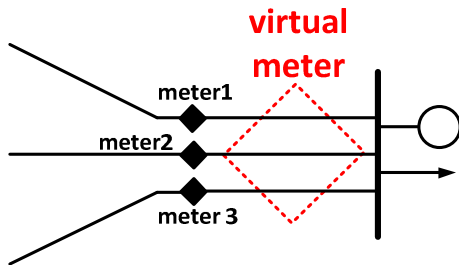


Fig. 1. Virtual meter of a bus with 3 lines

Considering virtual meters at buses of network, similar to Eqs. (3) and (4),  $Z^{*meas}$  and  $H^*$  can be introduced as follows.

$$\begin{bmatrix} \sum_{j \in K_1} f_{1j} \\ \sum_{j \in K_2} f_{2j} \\ \vdots \\ \sum_{j \in K_N} f_{Nj} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} \text{virtual meter 1} \\ \text{virtual meter 2} \\ \vdots \\ \text{virtual meter N} \end{bmatrix} = \begin{bmatrix} M_1^{VirMeas} \\ M_2^{VirMeas} \\ \vdots \\ M_N^{VirMeas} \end{bmatrix} \quad (10)$$

Equation (10) is similar to similar to Eq. (3). The left hand side of Eq. (10) is  $H^*$  matrix while the right hand side of Eq. (10) is  $Z^{*meas}$ . Therefore Eq. (10) is rewritten

as:

$$[H^*]X = Z^{*meas}; \quad H^* = \begin{bmatrix} \sum_{j \in K_1} f_{1j} \\ \sum_{j \in K_2} f_{2j} \\ \vdots \\ \sum_{j \in K_N} f_{Nj} \end{bmatrix}, \quad Z^{*meas} = \begin{bmatrix} M_1^{VirMeas} \\ M_2^{VirMeas} \\ \vdots \\ M_N^{VirMeas} \end{bmatrix} \quad (11)$$

where  $f_{ij}$  the flow of line connecting bus  $i$  to  $j$ , as written in Eq. 8(a) and  $M_i^{VirMeas}$  is the virtual measurement of the  $i^{th}$  bus as defined in Eq. (9). In fact, matrix  $H^*$  is network admittance matrix in which the row and column related to slack bus is deleted. Therefore,  $H^*$  is  $(N-1) \times (N-1)$  matrix which  $N$  is the number of network buses. Similar to Eq. (7), considering  $H^*$  and virtual measurements  $Z^{*meas}$ , the estimated value of  $X^*$  is:

$$X_{(N-1) \times 1}^{*est} = \left[ [H^*]_{(N-1) \times (N-1)}^T [R^*]_{(N-1) \times (N-1)}^{-1} [H^*] \right]^{-1} * [H^*]^T [R^*]^{-1} Z_{(N-1) \times 1}^{*meas} \quad (12)$$

In the proposed method,  $R^*$  is  $(N-1) \times (N-1)$  matrix. The variance of virtual meter of each bus is considered as equal to the maximum variance of meters connected to that bus. The advantages of the proposed method ( $H^*$  method) in comparison with the ( $H$  method) are:

- a) The dimension of  $H^*$  and  $R^*$  are independent of the number of transmission lines (or number of meters) and their dimensions are  $(N-1) \times (N-1)$ , ( $N$  is the number of network nodes). But the dimension of  $H$  and  $R$ , are directly related to the number of network lines (or number of meters).
- b) Lower memory space is needed to matrix  $H^*$  since  $H^*$  is  $(N-1) \times (N-1)$  matrix, while  $H$  is  $(N-1) * N_m$  and  $N_m > N-1$
- c) The calculation speed of computation is increased in the  $H^*$  method for the reason mentioned in item b.
- d) In the  $H$  method, both positive error and negative error in measurement, degrades the estimated values. However, in the proposed  $H^*$  method, a positive measurement error of a meter installed on a line connected to a bus (say meter1 in Fig.1), would be possibly neutralized by the negative measurement error of the other meter installed on the other line connected to that bus (say meter2 in Fig.1). Accordingly, the proposed method is more robust to measurement errors and thereby estimates the systems unknown variables more accurately than the previously presented  $H$  method. The flowchart of the proposed method is presented in Fig. 2.

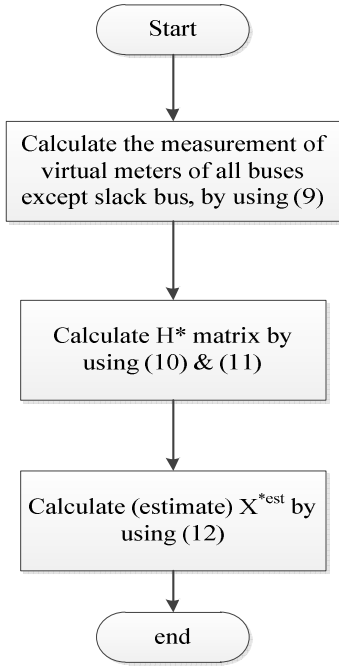


Fig. 2. Flowchart of the proposed method

4. CASE STUDY

In order to easily follow and understand the proposed method, at first, a 6-bus test system [1] is studied, and after that, IEEE 30-bus and IEEE 118-bus test systems are used to study the effectiveness of  $H^*$  method.

4.1. Simple 6-bus test system

A simple 6 bus test system with 11 transmission lines is shown in Fig. 3 [1]. Based on the proposed method, 11 measurements are needed to measure the flow of lines as shown in Fig. 3.

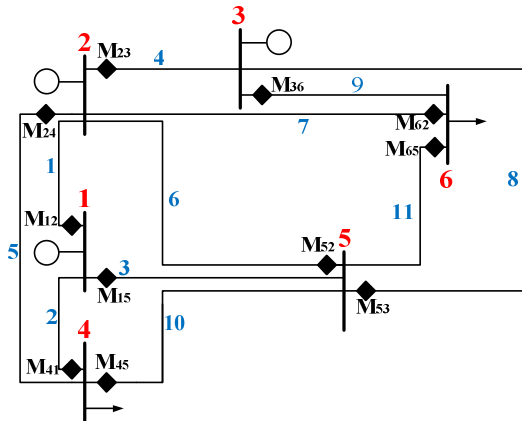


Fig. 3. Simple 6-bus test system [1]

Since the system lossless, only one meter is required to measure the active power of each line, not important whether it is installed at near end node or far end node of the interested line. Therefore, this matter does not cause any inaccuracy in the state estimation algorithm. The  $H^*$  matrix is made inspired by KCL. Therefore, the proposed

method is valid for both AC and DC state estimation. However, without loss of generality, in the case study, DC power flow is used, to easily follow the proposed method and compare it with  $H$  method taken in reference [1] of the paper.

The unknown variable being estimated ( $X$ ) is the phase angle of network buses voltage ( $X = \theta$ ). Bus 1 is considered as slack bus. Therefore, ( $x_1 = \theta_1 = 0$ ). The system load and generation data are taken in Table 1. Also  $S_{base} = 100$  MVA. Using DC power flow, the true values of system phase angles are reported in the last column of Table 1.

Table 1. 6-bus test system data and phase angle of buses (DC load flow)

Bus No.	$P_G$ (MW)	$P_D$ (MW)	Bus angle (degree)
1	100	0	0
2	50	0	-2.9024
3	60	0	-3.1679
4	0	70	-4.7632
5	0	70	-5.6902
6	0	70	-5.7418

The system lines and meters data are taken in Table 2. Also the true values of line flows, i.e., all meters are ideal, are shown in the last column of Table 2.

Using DC load flow equation, for meter M12 , the following equation will be written:

$$M_{12} = \frac{25.3}{100} = 0.253 \text{ p.u.} \Rightarrow \frac{\theta_1 - \theta_2}{x_{12}} = 0.253 \Rightarrow \frac{0 - \theta_2}{0.2} = 0.253 \Rightarrow -5\theta_2 = 0.253 \tag{13}$$

Writing the equation of all 11 meters, like that written for meter M12 in (13), the  $H$  matrix of this system will be obtained as follows [1]:

$$\begin{matrix}
 H \\
 \begin{bmatrix}
 -5 & 0 & 0 & 0 & 0 \\
 0 & 0 & 5 & 0 & 0 \\
 0 & 0 & 0 & -3.33 & 0 \\
 4 & -4 & 0 & 0 & 0 \\
 10 & 0 & -10 & 0 & 0 \\
 -3.33 & 0 & 0 & 3.33 & 0 \\
 -5 & 0 & 0 & 0 & 5 \\
 0 & -3.85 & 0 & 3.85 & 0 \\
 0 & 10 & 0 & 0 & -10 \\
 0 & 0 & 2.5 & -2.5 & 0 \\
 0 & 0 & 0 & 3.33 & -3.33
 \end{bmatrix}_{11 \times 5}
 \end{matrix}
 \times
 \begin{matrix}
 \begin{matrix}
 x \\
 \begin{bmatrix}
 \theta_2 \\
 \theta_3 \\
 \theta_4 \\
 \theta_5 \\
 \theta_6
 \end{bmatrix}_{5 \times 1}
 \end{matrix}
 \end{matrix}
 =
 \begin{matrix}
 \begin{matrix}
 Z^{meas} \\
 \begin{bmatrix}
 0.253 \\
 -0.355 \\
 0.331 \\
 0.018 \\
 0.325 \\
 -0.163 \\
 -0.248 \\
 -0.169 \\
 0.449 \\
 0.040 \\
 0.003
 \end{bmatrix}
 \end{matrix}
 \end{matrix}
 \tag{14}$$

It is noted that  $\theta_1$  is not included in Eq. (14), because it is a known value ( $\theta_1 = 0$ ). Since it is assumed that meter is ideal, the right hand side of Eq. (14) are the true value of lines flow, reported in the last column of Table 2. The system includes 11 lines and 5 unknown variables.

Accordingly, the size of  $H$  for this system is  $(11 \times 5)$ .

**Table 2. System lines and meters data and lines flows obtained from DC load flow**

Line No.	X (p.u.)	Meter ID	Near End bus	Far End bus	Line flow (MW)
1	0.20	M <sub>12</sub>	1	2	25.3
2	0.20	M <sub>41</sub>	4	1	-41.5
3	0.30	M <sub>15</sub>	1	5	33.1
4	0.25	M <sub>23</sub>	2	3	1.8
5	0.10	M <sub>24</sub>	2	4	32.5
6	0.30	M <sub>52</sub>	5	2	-16.2
7	0.20	M <sub>62</sub>	6	2	-24.8
8	0.26	M <sub>53</sub>	5	3	-16.9
9	0.10	M <sub>36</sub>	3	6	44.9
10	0.40	M <sub>45</sub>	4	5	4
11	0.30	M <sub>56</sub>	5	6	0.3

In order to obtain  $H^*$  and  $Z^*_{meas}$ , Eqs. (9) and (10) are used. Since bus 1 is the slack bus, then  $(x_1 = \theta_1 = 0)$ . Accordingly, the virtual meter should be considered at buses 2, 3, ..., 6 of the network. Consider bus 2 of network. Nodes {1, 3, 4, 5, 6} are connected to bus 2 of network. So based on Eq. (10), for bus 2 of network,  $i=2$  and  $K_2 = \{1, 3, 4, 5, 6\}$ . Considering Eq. (10), for bus 2 of the network, it will be as follows.

$$\frac{\sum_{j \in K_2} f_{2j}}{x_{21}(\theta_2 - \theta_1) + \frac{1}{x_{23}}(\theta_2 - \theta_3) + \frac{1}{x_{24}}(\theta_2 - \theta_4) + \frac{1}{x_{25}}(\theta_2 - \theta_5) + \frac{1}{x_{26}}(\theta_2 - \theta_6)} = \frac{M_2^{VM_{meas}}}{M_{21} + M_{23} + M_{24} + M_{25} + M_{26}} \quad 14(a)$$

$$\frac{1}{0.2}(\theta_2 - 0) + \frac{1}{0.25}(\theta_2 - \theta_3) + \frac{1}{0.1}(\theta_2 - \theta_4) + \frac{1}{0.2}(\theta_2 - \theta_5) + \frac{1}{0.3}(\theta_2 - \theta_6) = \frac{-M_{12} + M_{23} + M_{24} - M_{52} - M_{62}}{100} \quad 14(b)$$

$$\frac{27.3\theta_2 - 4\theta_3 - 10\theta_4 - 5\theta_5 - 3.33\theta_6}{100} = \frac{-25.3 + 1.8 + 32.5 - (-16.2) - (-24.8)}{100} = 0.5 \quad 14(c)$$

Using Eq. (10) for buses 3, 4, 5, 6 of network, the  $H^*$  matrix will be obtained as following equation:

$$\begin{bmatrix} 27.33 & -4.00 & -10.00 & -5 & -3.33 \\ -4.00 & 17.84 & 0 & -3.84 & -10.00 \\ -10.00 & 0 & 17.50 & -2.50 & 0 \\ -3.33 & -3.84 & -2.50 & 16.34 & -3.33 \\ -5.00 & -10.00 & 0 & -3.33 & 18.33 \end{bmatrix}_{5 \times 5} \times \begin{bmatrix} x \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}_{5 \times 1} = \begin{bmatrix} Z^*_{meas} \\ 0.5 \\ 0.6 \\ -0.7 \\ -0.7 \\ -0.7 \end{bmatrix} \quad (15)$$

Once the  $H$  and  $H^*$  of network is made, the voltage angle of network buses can be estimated by using Eq. (7) for  $H$  method and Eq. (12) for  $H^*$  method. In order to

evaluate the effectiveness of the  $H^*$  method, four cases are considered and the estimated values of  $H^*$  method are compared with those of  $H$  method.

Case 1: all meters are ideal

Case 2: all meters but M12 are ideal; M12=20 MW instead of 25.3 MW

Case 3: all meters but M53, M36 are ideal (M53 = -13 MW instead of -16.9 MW and M36 = 50 MW instead of 44.9 MW)

Case 4: all meters includes errors

The meter's measurements for case 1 and case 4 are taken in Table 3. The value of variance matrix for all meters are considered identical and equal to 0.0001 ( $\sigma = 0.01$ ). The results of these four cases are shown in Table 4. Since in case 1, all meters are ideal, the results of both  $H$  and  $H^*$  methods are equal to the true values of voltage angles of buses obtained by DC power flow which are reported in the second column of Table 4.

**Table 3. The measurement of meters in case 1 and case 4**

Line No.	Meter ID	Meter Measurement in Case 1 (MW)	Meter measurement in Case 4 (MW)
1	M <sub>12</sub>	25.3	27
2	M <sub>41</sub>	-41.5	-44
3	M <sub>15</sub>	33.1	32
4	M <sub>23</sub>	1.8	4
5	M <sub>24</sub>	32.5	30
6	M <sub>52</sub>	-16.2	-19
7	M <sub>62</sub>	-24.8	-22
8	M <sub>53</sub>	-16.9	-14
9	M <sub>36</sub>	44.9	40
10	M <sub>45</sub>	4	5
11	M <sub>56</sub>	0.3	1

The results of cases 2, 3, and 4 shows the superiority of  $H^*$  method to  $H$  method. As shown in the last row of Table 4, the maximum error of estimation in  $H^*$  method is lower than that of  $H$  method. In almost all of case studies, the error of  $H^*$  is lower than that of  $H$  method. However, in some rare cases, such as case 3, the  $H^*$  error is more than  $H$  method.

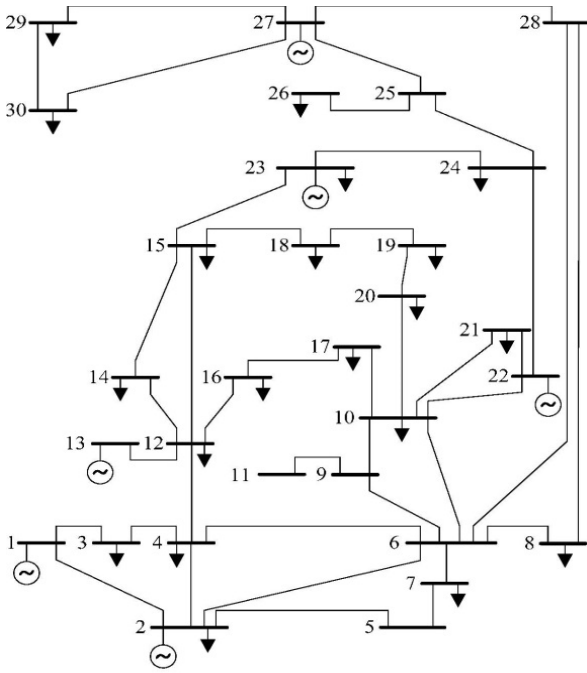
#### 4.2. IEEE 30-bus and IEEE 118-bus test systems

The IEEE 30-bus test system includes 30 buses and 41 transmission lines as shown in Fig. 4. Bus 1 is the slack bus, and therefore  $\theta_1$  is known and the other voltage angle of 29 buses of network should be estimated. Therefore,  $H$  is a  $(41 \times 29)$  matrix, while  $H^*$  is a  $(29 \times 29)$  matrix. The IEEE 118-bus test system includes 118 buses and 186 transmission lines. Bus 69 is the slack bus, therefore  $\theta_{69}$  is known and the other voltage angle of 117 buses of

network should be estimated. Therefore, for IEEE 118-bus system,  $H$  is a  $(186 \times 117)$  matrix, while  $H^*$  is a  $(117 \times 117)$  matrix. The results of case 4 (in which all meter includes measurement errors and  $\sigma = 0.01$ ) for these two systems are taken in Table 5.

**Table 4. The estimated and true value of system phase angle (degree) in  $H$  method and  $H^*$  method**

Bus No.	$\theta^{est}$ (degree)								
	Case 1		Case 2		Case 3		Case 4		
	$H$	$H^*$	$H$	$H^*$	$H$	$H^*$	$H$	$H^*$	
1	0	0	0	0	0	0	0	0	
2	-2.90	-2.90	-2.90	-2.61	-2.97	-2.92	-3.08	-3.12	-2.70
3	-3.17	-3.17	-3.17	-2.90	-3.21	-3.15	-3.43	-3.59	-2.84
4	-4.76	-4.76	-4.76	-4.53	-4.61	-4.77	-4.89	-4.87	-4.58
5	-5.69	-5.69	-5.69	-5.48	-5.70	-5.68	-5.88	-5.83	-5.41
6	-5.74	-5.74	-5.74	-5.48	-5.78	-5.74	-6.13	-5.85	-5.24
error%	0	0	10.15	2.39	3.46	8.29	13.55	10.12	



**Fig.4. IEEE 30-bus test system**

The results of Table 5 show that, in both test systems, the proposed  $H^*$  method reaches to more accurate solution with lower computation time with respect to  $H$  method. Especially, for IEEE 118-bus system, the computation burden of  $H$  method is more than twice of that of  $H^*$  method, indicating the advantage of  $H^*$  method in large size power system including hundreds of buses.

It is noted that the reported computation time in Table

5, includes both of making  $H$  matrix time and state estimation time (calculating  $X$ ).

**Table 5. Comparison of the  $H^*$  and  $H$  method results**

	Computation time		Maximum error (%)	
	$H^*$ method	$H$ method	$H^*$ method	$H$ method
IEEE 30-bus	0.0013	0.0015	18.53	19.42
IEEE 118-bus	0.0169	0.0350	6.43	9.60

In almost all of case studies, the error of  $H^*$  is lower than that of  $H$  method. However, in some rare cases, such as case 3 in Table 4, the  $H^*$  error is more than  $H$  method. However, it is rare that such case happens in real power systems. As reported, for IEEE 30 and IEEE 118 bus test system, the results of  $H^*$  method have lower error than those of  $H$  method. Accordingly, considering case 3 of Table 4, we should conclude that the error of  $H^*$  method usually (not always) is lower than that of  $H$  method. Also it can be concluded that,  $H^*$  method is usually more accurate than  $H$  method.

### 5. CONCLUSIONS

A more robust and accurate state estimation method is proposed in this paper, based on WLS method, by considering virtual meter for each bus, which is conceptually based on KCL law. By the suggested simple but applicable method, the state estimation algorithm is improved. As shown in the case studies, the proposed method is usually more insensitive to meter errors. The other salient feature of this method is its independency to the number of meters. It only depends on the number of networks buses. Furthermore, the proposed method yields more accurate results with lower computation burden, in comparison with the pervious method discussed in the paper. The capability of the proposed method would be better understood in real power systems including hundreds of transmission lines.

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