A Two-Stage Stochastic Programming Model for the Optimal Sizing of 
PV/Diesel/Battery in Hybrid Electric Ship System

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Abstract- Ships play the major role in bulk transportation and they need their special energy system. This paper proposes a stochastic programming method for optimal sizing of a hybrid ship power system with energy storage system (ESS), photovoltaic power (PV) and diesel generator. To account for uncertainties, in this study a two-stage stochastic mixed-integer non-linear programming is used to model the optimal design problem of the hybrid system for ships. The uncertainty of the hourly global solar irradiation and its effect on the output power of the PV system is taken into account. The probability density function of the global solar radiation follows a normal distribution. The Monte Carlo sampling approach is used to generate the scenarios with a specified probability and a proper scenario reduction method is used to decrease the computational burden of problem. Three cases are studied and the results are presented and compared.

Keyword: Monte Carlo method, Photovoltaic generation, Ship power system, Stochastic programming, Uncertainty.

1. INTRODUCTION

1.1. Motivation

One of the most important problems that today's world confront it, is the increase in the emission of the greenhouse gases such as CO₂. Increase in the amount of greenhouse gases directly affect the climate change, so it is necessary to control this event [1]. There are different sources of CO₂ emissions such as transportation especially merchant marine that is one of the most common ways to transport between countries and it plays a crucial role in the world economy. Although shipping only accounted for under 3% of total 13% of global CO₂ emission in 2012 [2]. It is estimated that the marine activity will be more than twice by 2050 [2, 3]. Considering the mentioned reasons, we tried to present a practical and suitable solution to design a hybrid ship power system using stochastic optimization approach, taking the cost minimization into account.

1.2. Literature review

Because of mentioned reasons, a lot of researches have been studied in this field. To reach this purpose there are 4 common ways as follow: 1) developing energy efficiency 2) using renewable energies such as wind or photovoltaic (PV) 3) using clean fuels to produce less carbon dioxide 4) using technologies that consume less fuel [3]. The main source of CO₂ emission in the ship power system is the diesel generator that should be replaced or improved by one of the 4 methods mentioned. Today cost of fuel accounts for more than half of shipping transportation cost and the bunker fuel price in comparison with 3 decades before is raised over 300%. Furthermore it will be a challenging situation for marine transportation [4-6]. To solve this problem, several hybrid PV/diesel/battery systems are designed and optimized by several researchers with different optimization methods and software. In [7-9] the investigation of hybrid power systems in ships have been studied. A lithium-ion battery combined with a diesel generator is studied in [9] for ship crane operation. In [10] an energy storage system (ESS) and all-electric ships have been used instead of bulk carriers to maximize fuel saving. In [11], optimal capacity of energy storage system is determined for a microgrid considering effects of reliability criterion and demand response program. The proposed system consists of thermal units, wind turbine and storage system. However, the sizing of renewable unit has not been envisaged in [11]. In [12], A hybrid Monte Carlo simulation-genetic approach (MCS-GA) is proposed to optimize the operation of battery energy.
storage systems and renewable energy generation systems in a typical distribution network. Moreover, the effect of uncertainties (electricity price and PVs output power) are modelled by Monte Carlo simulation method. In [13] an investigation of PV system characteristics on a moving ship has been proposed. Other researches have been done to increase battery life and consume less fuel with different control technologies in [8, 14]. The methodology proposed in [15] uses a dynamic programming model to determine the optimal operating strategy for a wind/diesel/battery system during 24 hours. The optimization of a hybrid wind/diesel/battery system based on the construction of trade-offs curves is presented in [16]. In [17] a non-linear constrained model has been used to optimize the mixed hydro/wind/solar/diesel/ESS system by an iterative method and the Quasi-Newton algorithm. A research to optimize the system cost and the CO$_2$ emission has been done in [18] using multi-objective evolutionary algorithm. Some authors presented stochastic optimization instead of deterministic optimization in their studies. In [19] a two-stage stochastic programming model is used to optimize the distributed energy systems. To solve the optimization problem, a two-stage decomposition based solution strategy is presented. In [20] a stochastic programming approach, for optimizing the hybrid power systems including the renewable resources and ESS with reliability analysis, is presented. A stochastic modeling framework is presented in [21] to solve the optimization problem with industrial customers and cogeneration facilities, conventional power production system, and heat only units. Reference [22] presented a stochastic model consisting of different components in a hierarchical manner, in space and time for significant wave height, a Bayesian hierarchical space-time model. To the best of this work the hybrid PV/diesel/battery ship power system is not extensively studied [23-26]. A method to reduce fuel cost using a PV system in a merchant marine vessel has been proposed in [23]. In [24] a stability test and economic evaluation of the hybrid PV/diesel ship power system has been presented. To decrease the emission of the ship at berth an evaluation is presented in [25]. In [27] the implementation of more than thirty energy efficiency and CO$_2$ emission reduction methods have been measured by a cross-sectional survey of ship owners and operators. Reference [26] presents a method to optimize the hybrid PV/diesel/battery ship power system to minimize the investment cost, fuel cost and the CO$_2$ emissions. In addition the integration of a considerable amount of PV into a ship power system to reduce the CO$_2$ emission is a challenging activity. The production of PV systems in ships depends on the solar ray's angle, longitude and latitude. The authors in [28] presented a different strategy of solar panel tilt angle optimization to improve the efficiency and the reliability of the system with no extra cost for the tracking. In [29-31] the prediction of performance, energy consumption, sizing curve, solar and wind energy parameters have been studied. An optimal sizing of hybrid PV/diesel/battery power system considering different tilt angles has been presented in [32]. In [33] a multi-objective genetic algorithm is presented to optimize fishing routes. The optimization is based on three parameters concluding resistance, seakeeping and stability to define the hull shape, optimizing hull offsets, length, beam and draft. As climate conditions have a strong effect on PV system performance and its output power, authors in [34] have presented a solution to formulate the correction factors for the output of PV modules in different date, local time, longitude and latitude. In [35, 36] an optimal size of the PV system has been proposed considering uncertainty and using real environmental conditions. Energy storage systems (ESSs) play an undeniable role in planning and management of hybrid energy systems. Therefore, it is necessary to model ESS in an appropriate way. Recently, many researches have focused on modelling of ESS in hybrid energy systems. Several types of batteries like as all-vanadium redox flow battery [37] and sodium sulfur (NaS) batteries [38] are developed for large scale energy storage applications. In [39] a sizing method for distributed battery energy storage system (BESS) in the distribution networks under high photovoltaic (PV) penetration level is presented. In order to gain insights into residential PV battery systems, a simulation model is developed in [40]. In [41] a multi-objective optimization problem is presented to minimize the overall ESS size and maximize the battery life time. Also, a model is presented to optimize the hybrid ESS (HESS) size and battery life time in electrical vehicles (EV).

### 1.3. Paper highlights

i. Solar irradiation is considered as stochastic process in the hybrid ship power system planning problem. This variable is forecasted using the Monte Carlo simulation approach.

ii. The load changing in a hybrid ship power system depends on the different operational conditions (regular cruising, full-speed sailing, docking, loading/unloading and anchoring)

iii. Sizing and performance assessment of ESS, especially state of charge/discharge, are investigated.
1.4. Paper organization
The remainder of the paper is organized as follows: Section 2 proposes the suggested stochastic method. Section 3 proposes the problem formulation. In Section 4 the case study, the evaluation and the simulation of this work is presented. Section 5 presents the conclusion.

2. STOCHASTIC FORMULATION
The fluctuation nature of solar irradiation leads to the uncertainty of the produced power by PV system. This uncertainty is modeled using the probability density functions (PDFs). Deterministic programming uses the determined group of inputs which results in the known group of outputs. On the other hand, the stochastic programming provides a better solution to plan under uncertainty. In the stochastic optimization problem, some or all of the parameters are probabilistic and it is divided into two-stage or multi-stage optimization [42]. Furthermore a stochastic based model is used in this paper to optimize the output power of hybrid system in a ship.

2.1. Scenario generation
To address the uncertainties a two-stage stochastic mixed-integer non-linear programming is used in this study. The Monte Carlo sampling method is used to generate the scenarios. In Monte Carlo simulation approach we allocate the probability \( p_\omega \) to each scenario that is the probability of occurrence. This high-speed sampling method hasn’t the limitation of the number of outputs at the same time. As the result the system size has no effects on the number of the samples for a specified accuracy level [43]. The scenarios are mixed to shape the scenario tree. The objective function is presented as the total net present cost of the fuel, emission and maintenance of diesel generator, PV and ESS. The financial indexes are the payback period and the interest rate in this study.

2.2. Scenario reduction
The number of scenarios in stochastic optimization is an effective index. So it is crucial to decrease them by an efficient scenario reduction method to decrease the calculation time and make the optimization practical in large-scale problems. On the other hand, the low number of generated scenarios makes the approximation unreliable. The reduction method provides an approximation with smaller number of scenarios and similar to the main system. Furthermore we determine an auxiliary group of scenarios and a probability measure based on this auxiliary group that is the closest to the main probability distribution. Efficient algorithms based on backward and fast forward reduction methods are presented that define optimal reduced measures. Backward and fast forward reduction methods are introduced and presented in [44, 45].

In this study, a tool called SCENRED for scenario reduction and modeling random data processes that is provided by the general algebraic modelling system (GAMS) is used to solve the problem. For large scenario reductions, the backward, fast forward and other methods are included in the SCENRED library. These methods in SCENRED construct an auxiliary group of prescribed cardinality or accuracy and allocate optimal probabilities to the specified scenarios[46].

3. PROBLEM FORMULATION

3.1. PV system
To design the PV subsystem, there are different methods that are proposed by several authors. In this paper a mathematical model is used to present the PV subsystem. The power produced by the PV panels can be calculated using the date, local time, time zone, longitude, latitude and temperature. The position of the ship in a specific hour has a determined irradiation, date and time. Furthermore the output power of each PV system at time \( t \) in season \( s \) and scenario \( \omega \) is estimated considering the solar irradiation by the following formula:

\[
P_{PV}(s,t,\omega) = \eta_{PV} \times S_{PV} \times G(s,t,\omega) \tag{1}
\]

Where \( \eta_{PV} \) is the PV generator efficiency at a specific time, \( S_{PV} \) is the area of the panels of the PV system \( (m^2) \) and \( G(s,t,\omega) \) is the solar radiance in season \( s \) at time \( t \) and scenario \( \omega \) \( (W/m^2) \). The PV generator efficiency at time \( t \) is calculated by the following formula [47]:

\[
\eta_{PV} = \eta_{PVr} \times \eta_{MPPT}(1 - \beta(Tc - Tcr)) \tag{2}
\]

Where \( \eta_{PVr} \) is the PV generator reference efficiency; \( \eta_{MPPT} \) is the efficiency of the power tracker equipment and considered 1 in this paper; \( Tc \) is the temperature of the PV panel \( (\circ\text{o}) \); \( Tcr \) is the PV panel reference temperature, which is considered 25\( \circ\text{o} \) in this paper, and \( \beta \) is the temperature coefficient of PV system efficiency, which varies from 0.004 to 0.006 \( \circ\text{o} \) for silicon panels and is taken 0.0048. The PV panel temperature is given by the following equation that is proposed by Markvar [48]:

\[
Tc = Ta + ((NCOT - 20) / 800)G(s,t,\omega) \tag{3}
\]
NCOT is the normal operating cell temperature and is 45°C herein. Ta is the ambient temperature which is taken 25°C in this paper. In this study hourly solar irradiation is modified by the mathematical method. The hourly solar radiances is calculated by Eq. (4).

\[
G(s,t,\omega) = G_{b}(s,t,\omega) + G_{d}(s,t,\omega) + G_{r}(s,t,\omega) = \\
G_{m}(s,t,\omega) \left[ \cos^{2} \left( \dfrac{\phi}{2} \right) \sin(\chi) + \cos(\theta) + \rho(\cos(\chi) + C)\sin^{2} \left( \dfrac{\theta}{2} \right) \right] + \rho(\cos(\chi) + C)\sin^{2} \left( \dfrac{\theta}{2} \right) \right]
\]

Where \( G_{b}(s,t,\omega) \) is the direct irradiation, \( G_{d}(s,t,\omega) \) denotes the sky diffuse radiation, \( G_{r}(s,t,\omega) \) is the ground reflected radiation and \( G_{m}(s,t,\omega) \) represents the direct normal irradiance on a surface perpendicular to the suns ray. The variables \( C, \rho, \chi \) denote the diffuse portion constant, the reflection index and the zenith angle, respectively. The variable \( \theta \) is the angle between the PV panel and the solar rays which is calculated by the following equation.

\[
\cos \theta = [\cos \phi \cos \chi + \sin \phi \sin \chi \cos (\xi - \zeta)]
\]

Where \( \phi \) represents the tilt angle from the horizontal surface which assumed to be 0° herein since the PV panels on the shipboard is horizontal. \( \xi \) and \( \zeta \) are the sun azimuth and plate azimuth angles, respectively; and is calculated using this formula [49]:

\[
\cos \theta = \cos \chi = \sin \delta \sin \lambda + \cos \delta \cos \lambda \cos \alpha
\]

Where \( \delta \) denotes the solar declination angle, which is calculated using (7). Variable \( \lambda \) is the latitude in degrees and \( \alpha \) represents the solar angle which can be determined by (8-13).

\[
\delta = 23.44 \sin \left[ \dfrac{360(d - 80)}{365.25} \right]
\]

\[
\alpha = \dfrac{360}{24} (LST - 12)
\]

\[
LST = LT + Tc / 60
\]

\[
Tc = 4(L_{loc} - LSTM) + EOT
\]

\[
LSTM = 15^{\circ} \tau_c \]

\[
EOT = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B)
\]

\[
B = 360(d - 81) / 364
\]

Where \( d \) is the number of the day, \( LST \) and \( LT \) represent the local standard time and the local time, respectively. \( EOT \) is the “equation of time” and accounts for the irregularity of the earth’s speed around the sun (minutes). The variable \( L_{loc} \) denotes the local longitude (degrees east> 0, and west< 0) and \( \tau \) is the difference between the current time zone and \( GMT \) (east> 0, west< 0). The output power of the PV system is bounded by (14).

\[
P_{PV(s,t,\omega)} \leq P_{PV} \leq P_{PV}^{max}
\]

### 3.2. Solar energy uncertainty

The hourly global solar irradiation has been presented by many researchers. References [50-53] indicate some of this studies. In this study we have used the stochastic characteristics of the solar radiation. The stochastic features of the global hourly solar radiation have been proposed by [54]. Their studies terminated to obtain these results:

1. The fluctuation nature of the global irradiation in winter is stronger than summer.
2. The fluctuation nature of the solar irradiation in morning and late afternoon is stronger than solar noon.
3. The probability density function of the global solar radiation follows a normal distribution. However, the eigenvalues vary in different times.

In this study the probability distribution of the hourly solar irradiation is taken normal distribution according to [54]. Table1 demonstrates the details. Where \( \mu \) is the value of each point in solar irradiation profile and \( \sigma \) is the standard deviation. The flowchart of scenario generation procedure for solar irradiation has been presented in Fig.1.

<table>
<thead>
<tr>
<th>Month</th>
<th>Hour</th>
<th>Distribution</th>
<th>Standard deviation (( \sigma ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>November,</td>
<td>9:00 am to 3:00 pm</td>
<td>( N(\mu, \sigma^2) )</td>
<td>12% ( \mu )</td>
</tr>
<tr>
<td>April</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>November,</td>
<td>The remainder of the day</td>
<td>( N(\mu, \sigma^2) )</td>
<td>25% ( \mu )</td>
</tr>
<tr>
<td>April</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May, October</td>
<td>9:00 am to 3:00 pm</td>
<td>( N(\mu, \sigma^2) )</td>
<td>3% ( \mu )</td>
</tr>
<tr>
<td>May, October</td>
<td>The remainder of the day</td>
<td>( N(\mu, \sigma^2) )</td>
<td>8% ( \mu )</td>
</tr>
</tbody>
</table>
3.3. Diesel generator

In case that the total power produced by the PV and ESS is insufficient, the diesel generator accounts for the load demand. Furthermore, the diesel generator is a necessary component in a hybrid system. One of the most important parameters in the diesel generator is the fuel consumption that should be evaluated. The fuel consumption of the diesel generator in season $s$ at time $t$ and scenario $\omega$, $FC(L/h)$, depends on the output power and is calculated as [55]:

$$FC(s,t,\omega) = a \times P_{\text{Diesel}}(s,t,\omega) + b \times P_{\text{ref}}^\text{Diesel}$$  \hspace{1cm} (15)$$

Where $P_{\text{ref}}^\text{Diesel}$ is the reference power; $P_{\text{Diesel}}$ is the produced power of the diesel generator. The factors $a$ and $b$ are the coefficients of the consumption curve. The output power of the diesel generator should satisfy (16).

$$P_{\text{Min}}^{\text{Diesel}} \leq P_{\text{Diesel}}(s,t,\omega) \leq P_{\text{Max}}^{\text{Diesel}}$$  \hspace{1cm} (16)$$

Where $P_{\text{Min}}^{\text{Diesel}}$ and $P_{\text{Max}}^{\text{Diesel}}$ are the minimum and maximum output power of diesel generator, respectively.

3.4. ESS system

In order to reach the considered output power, ESS system regulates the PV panels output by the mean of charge and discharge of the battery. ESS system regulates the output power by absorbing or generating energy. Furthermore, it is noteworthy that charge/discharge of the ESS is modelled in the second stage of proposed stochastic programming.

**Charge:** In case that the total power generated by the PV panels and the diesel generator exceed the demand, the charging operation takes into account and store the extra power.

**Discharge:** In case that the total generated power deficit the considered amount ESS system injects energy and compensates the power demand. This performance is provided by the discharge operation of the battery.

**Storage sizing:** The ESS system stores or re-distributes the energy generated by PV panels and diesel generator subjected to its power and energy rating.

The output power of the PV system has a fluctuating nature as the result in this study a LiFePO$_4$ battery is used as an ESS. In case that the total power exceeds or deficit the demand and the total power produced by the diesel generator and the PV system cannot satisfy the load, the battery takes into account. Equation (17) defines the energy of the ESS in season $s$ at time $t$ and scenario $\omega$:

$$E(s,t+1,\omega) = (E(s,t,\omega) \times \eta_{\text{in}}) + (E_{\text{in}}^{\text{input}}(s,t,\omega) \times \eta_{\text{battery}}) - \frac{E_{\text{out}}^{\text{output}}(s,t,\omega)}{\eta_{\text{battery}}}$$  \hspace{1cm} (17)$$

The energy stored in each battery is limited by (18).

$$E(s,t,\omega) \leq E_{\text{ESS}}$$  \hspace{1cm} (18)$$

Where $E_{\text{ESS}}$ is the maximum energy capacities of the battery. Equations (19) and (20) define the minimum and maximum bounds of the input, $E_{\text{input}}^{\text{input}}$, and output, $E_{\text{input}}^{\text{output}}$, power of the battery, respectively:

$$E_{\text{input}}^{\text{input}}(s,t,\omega) \leq \frac{E_{\text{in}}^{\text{input}}(s,t,\omega) \times E_{\text{Max}}^{\text{input}}}{\eta_{\text{battery}}}$$  \hspace{1cm} (19)$$

$$E_{\text{input}}^{\text{output}}(s,t,\omega) \leq \frac{E_{\text{out}}^{\text{output}}(s,t,\omega) \times E_{\text{Max}}^{\text{output}}}{\eta_{\text{battery}}}$$  \hspace{1cm} (20)$$

![Fig. 1. Flowchart for solar irradiation scenario generation process.](image-url)
\[ E_{\text{output}}(s, t, \omega) \leq \zeta_{\text{output}}(s, t, \omega) \times E_{\text{Max}} \]  

(20)

The battery cannot both charge and discharge in a same time. Equation (21) defines this:

\[ \zeta_{\text{input}}(s, t, \omega) + \zeta_{\text{output}}(s, t, \omega) \leq 1 \]  

(21)

Where \( \zeta \) is a binary variable for charge and discharge state. The ESS charge-discharge scheme is shown schematically in Fig. 2.

3.5. Load model

To design an efficient and a reliable hybrid ship power system, the features of the load demand profile are important parameters. Table 2 illustrates the total load demand in five operating situations of the ship, which are specified in details in Table 3. The five different load demands are 1580 kW for regular cruising, 1790 kW for full-speed sailing, 1650 kW for docking, 1290 kW for loading/unloading and 500 kW for anchoring. Consequently, the ship stops in 6 cities that are Dalian in China, Shanghai in China, Hong Kong in China, Matara in Aden, Singapore, and Dalian. The peak load is 1790 kW and the off-peak load is 500 kW.

<table>
<thead>
<tr>
<th>Load condition</th>
<th>Regular cruising</th>
<th>Full-speed sailing</th>
<th>Docking</th>
<th>Loading and unloading</th>
<th>Anchoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (kW)</td>
<td>1580</td>
<td>1790</td>
<td>1650</td>
<td>1290</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 2. Different loads of ship operating system.

<table>
<thead>
<tr>
<th>Load condition</th>
<th>Duration of different ship load conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dalian</td>
<td>2 6 4</td>
</tr>
<tr>
<td>Shanghai</td>
<td>2 8 0</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>2 14 4</td>
</tr>
<tr>
<td>Singapore</td>
<td>2 12 5</td>
</tr>
<tr>
<td>Matara</td>
<td>2 7 6</td>
</tr>
<tr>
<td>Aden</td>
<td>2 6 4</td>
</tr>
</tbody>
</table>

Table 3. Duration of different ship load conditions

<table>
<thead>
<tr>
<th>Life span (year)</th>
<th>Investment cost ($/kW)</th>
<th>Replacement cost ($/kW)</th>
<th>Efficiency (%)</th>
<th>Length/width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1800</td>
<td>1500</td>
<td>17</td>
<td>1.660.99</td>
</tr>
</tbody>
</table>

Table 4. Parameters of the PV system

3.6. Objective function

The objective function of the hybrid ship power system is to minimize the cost function, meet the operational constraints and consisting of the fuel, emission and maintenance cost of the diesel generator, the capital cost of PV panels and ESS. The objective function is described in (22):

\[
\text{Min } \sum_{s=1}^{NS} \sum_{t=1}^{NT} \sum_{\omega=1}^{NW} \left[ FC(s, t, \omega) + EC(s, t, \omega) + MC(s, t, \omega) \right] + PVC \times \tau_{PV} + ESSC \times \tau_{ESS}
\]

(22)

Where \( \pi_{s, t, \omega} \) is the possibility of a specific scenario, \( FC \) is the fuel cost, \( ES \) is the emission cost, \( MC \) is the maintenance cost of diesel generator and \( \tau \) is the capital recovery factor for turning the initial value to an annual capital cost, which are presented by the following equations:
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\[ FC(s,t,\omega) = PF \times (a \times P_{\text{diesel}}(s,t,\omega) + b \times P_{\text{ref}}^{\text{diesel}}) \]  \hfill (23)

\[ EC(s,t,\omega) = PE \times (a \times P_{\text{diesel}}(s,t,\omega) + b \times P_{\text{ref}}^{\text{diesel}}) \]  \hfill (24)

\[ MC(s,t,\omega) = PM \times (a \times P_{\text{diesel}}(s,t,\omega) + b \times P_{\text{ref}}^{\text{diesel}}) \]  \hfill (25)

\[ \tau = \frac{i(1+i)^n}{(1+i)^n - 1} \]  \hfill (26)

where, \( PF, PE \) and \( PM \) are the fuel, emission and maintenance prices for the diesel generator, respectively. \( i \) is the interest rate and \( n \) denoting number of the years of life span for PV system or ESS. \( PVC \) is the installation cost of the PV panels and can be formulated as follows:

\[ PVC = (COST_{\text{PV}}^{\text{ref}} + COST_{\text{PV}}^{\text{rep}}) \times P_{\text{PV}} \]  \hfill (27)

where, \( COST_{\text{PV}}^{\text{ref}} \) and \( COST_{\text{PV}}^{\text{rep}} \) denote the reference installation cost and replacement cost of PV system, respectively. \( ESSC \) is the installation cost of the ESS and can be formulated as follows:

\[ ESSC = (COST_{\text{ESS}}^{\text{ref}} + COST_{\text{ESS}}^{\text{rep}}) \times E_{\text{ESS}} \]  \hfill (28)

where, \( COST_{\text{ESS}}^{\text{ref}} \) and \( COST_{\text{ESS}}^{\text{rep}} \) are the reference installation cost and replacement cost of LiFePO\(_4\) battery, respectively.

Finally, in addition to mathematical model of ship power system components, energy balance has to be included in the optimization as follows:

\[ P_{\text{diesel}}(s,t,\omega) + P_{\text{PV}}(s,t,\omega) + E_{\text{input}}(s,t,\omega) \]  \hfill (29)

\[ -E_{\text{output}}(s,t,\omega) = P_{\text{Load}}(s,t,\omega) \]

4. CASE STUDY

In this study a two stage mixed-integer nonlinear stochastic programming is used to optimize the sizing problem of a hybrid ship power system, as presented in Fig. 3. The effect of using the uncertainty of the solar power, different loading conditions and ESS in the ship is evaluated. The coefficients of the consumption curve, emission and maintenance of diesel generator are obtained from [26, 55, 56], respectively. The parameters of the PV and the ESS are presented in Table 4 and Table 5, respectively [38, 57, 58].

4.1. Ship parameters and route detail

The aim of this work is to find an optimal sizing of a hybrid PV/diesel/ESS ship power system with taking the uncertainty of the solar irradiation and the PV system output into account. Data of this work is based on a practical project named "study on the application of photovoltaic technology in the oil tanker ship" in china [26]. The total area of PV modules is 2000\( m^2 \) and the
The maximum weight of the oil tanker is considered 100,000 tons. The length, width and height of this oil tanker are 332.95m, 60m and 30.5m, respectively.

The cost and emissions of hybrid ship power system have been evaluated in this work. The main components of the system are a generating PV array, a diesel generator and an ESS. Due to the stand-alone mode of ship power system the diesel generator must have the ability to supply the whole load demand.

4.2. Correction coefficient for PV system

The PV system is one of the main parts of the hybrid ship power system so it has a considerable effect on the optimization and simulation. The output power of the PV system is related to the solar irradiation. Furthermore it is important to evaluate the correction coefficient for the solar irradiation. Equations (4-13) are used to model the correction coefficient in various seasons and hours. The results are shown in Fig. 4.

In the summer (day 40 to 80) the hourly global solar irradiation is stronger as the result the correction coefficient will be larger in this season as shown in Fig. 3. Furthermore the irradiation in summer (July to September) has a deep impact on the output power of the PV system and the optimization problem. The highest value of solar irradiation along the sailing route, based on data from the GeoModel Solar Company [59], is on April 19th.

4.3. Results and discussion

In order to solve the stochastic planning problem of hybrid ship power system optimization, 1000 scenarios for the solar radiation are generated by the Monte Carlo sampling method. All of the probabilities of each scenario considered equal in this paper(1/1000) [43, 60].

As mentioned in 2.2 we use the GAMS/SCENRED program for scenario reduction by using the fast back forward method. The probability of each remaining
scenario is indicated in Table 6. Considering all above assumptions, the proposed MINLP problem is implemented in GAMS 23.6 software and is solved using the SBB solver [61] on a PC with an Intel Core Duo 2.2 GHz processor and 4 GB of RAM.

Three different cases are conducted to illustrate the effectiveness of proposed sizing method for the ship power system. In case 1, no PV array and ESS are considered. In case 2, in addition to the diesel generator, PV arrays are included in the ship power system sizing problem. Furthermore, in case 3, ESS is considered in the sizing problem. The results are presented in Table 7.

As shown in Table 7, in case 2, by using the PV arrays, total ship power system cost decrease significantly in comparison with case 1. In addition, due to the application of PV arrays, fuel consumption and emission of the diesel generator are reduced. As it is clear, in case 3, by considering ESS in the ship power system problem, with respect to the case 2, total ship power system cost decreases. Moreover, in case 3, by using the ESS, it can be found that PV size and PV installation cost are increased but total cost of design is decreased with respect to them in case 2.

On the other hand, the uncertainty of solar irradiation is one of the key parameters affecting the ship system configuration. Table 8 illustrates how the fluctuation nature of solar irradiation impact the total sizing cost and configuration of ship system. As shown in Table 8, when the solar irradiation assumed to be intermittent the total cost of planning increases from by about 16%. Moreover, as presented in Table 8, the fuel and emission costs decrease by about 5%. These lie in the fact that considering uncertainty of solar irradiation needs more PV panels and storage capacity.

5. CONCLUSIONS
In this study a two-stage stochastic mixed integer non-linear programming method is presented to find the optimal sizing of the hybrid PV/diesel/battery ship power system with respect to the uncertainty of the solar irradiation and output of the PV system. The optimizations by the stochastic programming method are more reliable. The optimization problem is solved considering the five different load conditions (regular cruising, full-speed sailing, docking, loading/unloading and anchoring) and the navigation route which is from Dalian in China to Aden in Yemen. The solar irradiation has fluctuation nature and uncertainty of that is taken into account. This method is used to minimize the cost function, meet the operational constraints and consisting of the fuel cost, emission cost and the maintenance cost of diesel generator, the capital cost of PV panels and ESS. The numerical results of 3 case-studies are presented and discussed. In addition, by considering the solar irradiation uncertainty in the planning model, the total cost of planning increases by about 16% and the fuel and emission costs decrease by about 5% compare to the case which neglects this uncertainty. As future work, the impact of other uncertainties like temperature, load and the random outages of components on sizing of hybrid ship system can be studied.

| Table 8. Configuration and economic comparison for stochastic and deterministic cases |
|----------------------------------|-----------------|-----------------|
| Ship power system                | Case 3          | Case 3          |
|                                  | (Stochastic)    | (Deterministic) |
| Total cost ($)                   | 1825594         | 1571000         |
| Fuel cost ($)                    | 1458785         | 1533400         |
| Emission cost ($)                | 299484          | 314802          |
| Maintenance cost ($)             | 15360           | 16145.64        |
| PV net present cost ($)          | 44013           | 21646           |
| PV size (kW)                     | 305             | 150             |
| ESS net present cost ($)         | 979             | 609             |
| ESS size (kWh)                   | 291             | 181             |

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