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Presenting a New Method Based on Branch Placement for Optimal Placement of Phasor Measurement Units

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ABSTRACT
In this paper, a new method based on branch placement for the optimal positioning of Phasor Measurement Units (PMUs) in power systems is proposed. In this method, the PMUs are in type of single-channel and are installed at the beginning of the branches. Therefore, they are able to measure the bus voltages. Also, the installation of the PMUs on the branches increases the security of observability in comparison to the bus placement method at the time of line or PMU outages. In this paper, the Genetic Algorithm (GA) method is used to solve the optimization problem. The proposed method is applied to IEEE 30-bus and 24-bus case study networks. In addition, the method is tested on a real 400 kV network in Iran.

KEYWORDS: Genetic algorithm (GA), Monitoring, Optimal placement, Phasor Measurement Unit (PMU), Power system

1. INTRODUCTION

The emergence of Phasor Measurement Units (PMUs) has created a great revolution in the operation of power systems. The main application of the PMUs includes real time monitoring and controlling of power systems. One of the most important components of modern energy management systems is the process of estimating the states of the power system based on real time measurements [1].

The state estimation technology was created in 1960. This technology was based on non-synchronized measurement quantities [2]. To estimate the states of a system, a large number of non-linear equations should be solved synchronically [3]. However, the provided technology was not able to accurately assess the dynamic status of power system due to low-rate and low-speed measurements [4]. One of the appropriate solutions for the real time monitoring of power systems is the use of PMUs. The voltage and current waveforms of widely dispersed PMUs are synchronized with respect to a global positioning system (GPS) clock [5].

Due to the high cost of PMUs, it is not possible to install them at all buses; therefore, one of the important problems for a given network is to minimize the number of PMUs while maintaining the network observability [6,7].

Various methods have been proposed for the optimal placement of PMUs on the buses to minimize the cost and increase the total network observability [8]. The Genetic Algorithm (GA) has been used to solve the PMU placement problems [9]. In [10,11], the Binary Particle Swarm Optimization (BPSO) algorithm and in [12,13] binary linear programming were used. In [14], optimal PMU placement is performed using the branch placement method. But in the objective function of [14], the installation cost is not considered. As regards, the number of PMUs via

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branch placement method has been constant size ($\frac{1+N}{2}$) which N is number of system buses.

The PMUs which have a limited channel capacity and just measure sending end voltage and sending end current are called branch PMUs [15]. Although the presented methods solve the placement problem with high precision, they have a low visibility security at the time of line or PMU outages.

In this paper, a new method is presented to find the optimal placement of the PMUs based on the branch placement. Therefore, the PMUs are installed on branches near the buses, instead of being placed on the buses. The proposed method is so flexible that can keep the observability of the network in the case of probable line or PMU outage. The important advantages of the proposed method are as follows:

1) Increasing the security of the observability of power system.

2) Reducing the PMU installation costs associated with the proposed method against the methods of bus placement.

In this paper, the optimal placement problem is also solved with Harmony Search (HS), Imperialistic Competition (IC), Simulated Annealing (SA), and GA. The results of SA versus the results of HS, IC and GA algorithms are not optimal. The results of the HS, IC and GA algorithms are similar.

2. OBSERVABILITY PROBLEM

Power system observability means the calculation of the network variables to estimate the states of the system [1]. The current phasors of the branches connected the buses and the voltage phasor are often considered as network variables. In general, the observability is divided into two categories; the numerical and the topological [2]. In the numerical category, the mathematical model is used for the power system and its measurement. The topological category is based on the following principles [16]:

1. If the voltage of a bus and the currents of the branches connected to that bus are visible; the voltage of the bus at the other side of the branches can be calculated.
2. If a PMU is installed on a bus, the voltage phasor of the bus is known. Also, the branches connected to the bus have obvious currents.
3. If the voltages of the buses at both ends of a branch are known, the branch current is obtained by "KVL" rule.
4. If the currents of all branches leading to the zero injection bus are known except one, the unknown current phasor can be calculated by "KCL" rule.

In this paper, the topological methods are used to analyze the observability of the power system in which the PMUs are installed on the branches instead of being installed on the buses. The second and third principles of observability are therefore amended as follows:

- The current phasor of the branches in which the PMUs installed is known. Also, the bus voltage connected to the mentioned branch is known.
- If the branch current is known, the voltages of the buses at both ends of the branch are obtained by "KVL" rule.

PMU-based observability means that the system is observable with the minimum number of the PMUs and is able to send the information related to the network status moment to moment to the control center. In this paper, the observability security index is given as follows:

$$E = \frac{r}{N} \quad (1)$$

where, N is the number of the buses in system, r is equivalent to the number of the buses which is still observable while one PMU or one branch line is out of service. This factor is called measurement increase factor. To increase the observability, E should be close to its maximum value.

3. THE PROPOSED METHOD FOR PMU PLACEMENT

The placement of the PMUs on the buses has been performed using the intelligent algorithms [1-6]. In these methods, the PMUs are installed on the buses which are previously selected in the system. These units measure the bus voltages and the currents of all branches that connect to these buses. In the proposed method, the PMUs are installed on the branches. In this method, the PMU measures the current of the branch in which is installed and the bus voltage connected to the regarded branch.

Consider a power system including l lines. The placement objective function is as follows:

$$min \sum_i^l C_i . X_i$$
$$B = A.X$$
$$\text{s.t.} \quad B \geq \hat{I} \qquad (2)$$

where, l is the number of system branches; X_i is the binary variable which is 0 or 1; if $X_{i}=1$ a PMU is installed on the i^{th} branch, and if $X_{i}=0$ no PMU is installed on the i^{th} branch. C_i is the cost of PMU installation on the branch i; A is the matrix of branch connections to the buses that is defined as follows:

$$A_{ij} = \begin{cases} 1 & if\ branch\ j\ is\ incident\ to\ bus\ i \\ 0 & otherwise \end{cases}$$

B is the observability matrix of the system. ($B \geq \hat{I}$) is constraint of optimal placement problem; the number of observability should be greater or equal to one (≥1).

Also, $\hat{I}=[1\ 1\ 1 \ldots 1]^T$.

To describe the proposed method, a 5-bus sample network of Fig.1 is used.

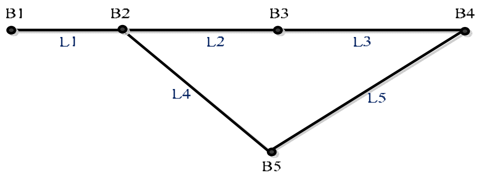

Fig. 1. Sample 5-bus system

The matrix of connection to the bus for the network is as follows:

$$A = \begin{bmatrix} 1\ 0\ 0\ 0\ 0 \\ 1\ 1\ 0\ 1\ 0 \\ 0\ 1\ 1\ 0\ 0 \\ 0\ 0\ 1\ 0\ 1 \\ 0\ 0\ 0\ 1\ 1 \end{bmatrix} \qquad (3)$$

Solving the optimization problem of the sample network using 2, one of the probable responses will be as follows:

$$X=[1\ 0\ 1\ 1\ 0\ 0]^T \qquad (4)$$

The installation cost of the PMUs varies based on the number of the channels. One of the advantages of this method is that since all of the installed PMUs are single channel type, the PMU and the installation costs are less than the conventional methods. To show how the number of the channels affects the PMU cost, Table 1 is provided from [15].

Table 1.PMU cost with respect to the channels

Number of channels	Cost of PMU($)
1	1500
2	2000
3	2500
4	3000
5	3500
6	4000
7	4500
8	5000

In this paper, the placement is performed by incorporating the costs of PMUs with respect to the number of the channels into the objective function. Therefore, to obtain better results, a novel objective function is proposed as follows:

$$min\ OF = \sum_i^l C_i X_i . - k \sum_p^n \sum_{q=1}^m [a_{pq}] \qquad (5)$$

where, k is a constant coefficient. $[a_{pq}]$ represents the element of matrix B at row p and column q. In Eq. (5), the first part of the OF shows the cost of the installed PMUs and the second part of the OF shows the number of the system observability according to the constraints ($\text{s.t.} \quad B \geq \hat{I}$).

4. GENETIC ALGORITHM

GA is one of the earlier variants of Evolutionary Algorithms (EAs) developed by Holland in 1975 [17]. The global solutions can be found for both linear and nonlinear formulations. The optimal solution searching process is independent of the form of the objective function, and will not be trapped in the rapid descending direction introduced by local minima. In other words, the solution of a complex problem can be started with weak initial estimations and then can be corrected in evolutionary process of fitness. GA generally includes three fundamental genetic operators of reproduction, crossover, and mutation. These operators conduct the chromosomes toward better fitness. In this paper, the binary GA is used to solve the optimization problem.

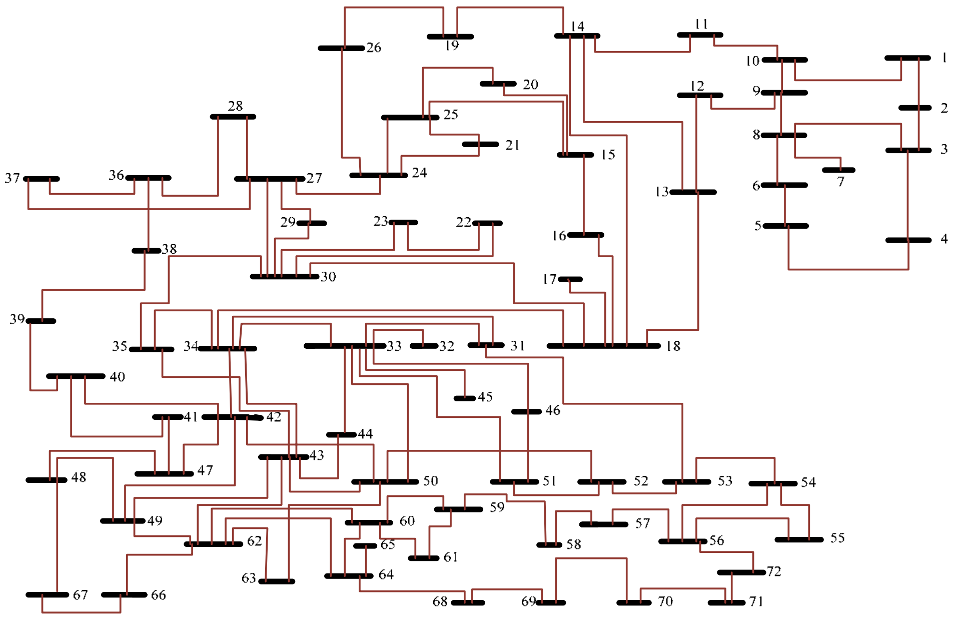

Fig. 2. Single-line diagram of network 400 kV in Iran

5. SIMULATION RESULT

A. Case studies:

The proposed method is applied to the IEEE 24-bus and 30-bus systems described in [18]. Also to show the performance of the proposed method, Iran 400 kV system is considered as the test system. The genetic algorithm is used for optimization in these simulations. In Table 2, the number of the branches of the test systems is given. Figure 2 shows the single-line diagram of 400 kV network of Iran [18].

In this paper, according to the average cost of all the PMUs and system observability, the *k* constant of (5) should be 1150 for both of branch placement and bus placement.

Table 2. The branches of test systems

Network	Number of branches
24 buses IEEE	34
30 buses IEEE	41
400 kV of Iran	103

B. Simulation results

Simulation process is provided in three steps:

Step 1: In the first step, the simulation is performed based on the conventional methods (bus placement methods). The methods are applied to the three case studies and the PMUs installation costs are calculated based on the number of channels. In table 3, the first column shows the case study; the second column shows the number of the PMUs resulting from the optimization problem; the third and the fourth columns also illustrate the location of the PMUs and their total cost, respectively.

According to Table 3, for example, a PMU is installed on the bus number 18 in the 400 kV network of Iran. With respect to Fig. 2, this PMU as 6 channels because there are 6 lines connected to the bus number 18; and according to Table 1, its cost is equal to 4000$. As another example, the PMU is installed on bus number 33 which has 8 channels; therefore, its cost is 5000$. According to Table 3, the cost of the PMU placement in the 400 kV network of Iran is calculated as follows:

The buses 1, 5, and 70 have two channels so each of them costs 2000$. The buses 13, 36, 40, 48, 51, 54, and 59 have three channels and each costs 2500$. The buses 8, 14, 24, 25, 56, and 64 have four channels and each costs 3000$. Similarly, the cost of each PMU on the buses 18, 30, and 62 is 4000 $, and the cost associated with PMU of the bus number 33 is 5000$. Therefore, the total cost for the network of Iran is 58500$.

Step 2: In this step, the branch placement method is investigated through the case studies. The obtained results are shown in Tables 4-6. All of the installed PMUs are single channel. Therefore, the cost of each installed PMU is 1500$ according to Table 1. The second column of Tables 4-6 shows the number of the installed PMUs. Regarding the fact that the costs of all PMUs are the same (1500 $), the total cost of the installed PMUs are calculated in the fourth column of the Tables 4-6.

In the third column of Tables 4-6, the positions of installed PMUs are given. The branches are shown in this column by the sending and the receiving bus numbers. It is noted that the PMUs are installed next to the sending bus of the branch. For example, in 5th row of Table 4, a PMU should be installed on the branch between buses 7 and 8. It is noted that the PMU should locate next to the sending bus (bus 7).

Table 3. The number and location of PMUs based on bus placement method

Network	Number of PMUs	Location of PMUs	Cost($)
24 buses	7	2,3,8,10,16,21,23	19000
30 buses	10	3,5,8,10,11,12,19,24,25,30	24500
400 kV of Iran	20	1,5,8,13,14,18,24,25,30,33,36,40,48,51,54,56,59,62,64,70	58500

Table 4. The number and location of PMUs according to the branch placement method for a 24-bus system

Network	Number of PMUs	Location of PMUs		Cost($)
		sending bus No.	receiving bus No.	
24 buses IEEE	12	1	3	18000
		2	6	
		4	9	
		5	10	
		7	8	
		11	13	
		12	23	
		14	16	
		17	18	
		19	20	
		21	22	
		24	15	

Step 3: In this step, the results of associated with the step 1 and step 2 are compared. The step 1 is based on the bus PMU placement method whereas the second step is based on the proposed branch PMU placement method. A brief picture of the simulations results is presented in Table 7. As shown in this Table, the branch placement method has less installation cost as compared to the bus placement method. For example, the cost reduction in 24-bus, 30-bus, and Iran network case studies is about 5.3%, 8.3%, and 5.1%, respectively.

Table 5. The number and location of PMUs according to the branch placement method for a 30-bus system

Network	Number of PMUs	Location of PMUs		Cost($)
		From bus No.	To bus No.	
30 buses IEEE	15	2	1	22500
		4	3	
		5	7	
		6	8	
		9	11	
		10	20	
		12	13	
		14	15	
		16	17	
		18	19	
		21	22	
		23	24	
		25	26	
		27	28	
		29	30	

Here, the visibility security index is calculated. Therefore, the possibility of the conditions such as a line outage and a PMU outage is considered (Table 8). The 2nd column of Table 8 shows various states of PMU and line outages in bus placement method. In 5th column, these states are presented for branch placement method. Then factor *r* is calculated. This factor indicates the number of buses which are still observable when one PMU or branch is eliminated. The value of *r* in columns 3 and 6 of the Table 8 is obtained in different possible conditions. Finally, observability security index (*E*) is obtained using (1). The calculated index *E* is presented in columns 4 and 7 of Table 8 for the three case studies.

For a better comparison between observability security in the proposed method and conventional method, the mean observability security index associated with the 24-bus and 30-bus, and Iran 400 kV networks is presented in Fig. 3. This chart shows that the observability security index associated with the branch placement method in all states is higher than those calculated for the bus placement method. The obtained results show that the branch placement method is much better than the bus placement method in terms of cost and visibility security.

Also, the proposed method is compared to the method presented in [14]. Although the method of [14] is based on the branch placement method, the economical constraints have not been incorporated in the objective function. To study the effects of this ignorance, the objective function of [14] is applied to IEEE 24-bus, 30-bus systems and Iran 400 kV system and the results are presented in Table 9.

The results of Table 9 show that in the systems with a few buses (e.g. 24-bus and 30-bus) the results of the two methods are the same. The advantage of the proposed approach is evident in the networks with a large number of buses.

Table 6. The number and location of PMUs according to the branch placement method for Iran 400kV system

Network	Number of PMUs	Location of PMUs		Cost($)
		From bus No.	To bus No.	
400 kV of Iran	37	1	2	55500
		3	4	
		5	6	
		8	7	
		9	10	
		12	13	
		14	11	
		15	16	
		18	17	
		19	26	
		22	23	
		24	21	
		25	20	
		27	28	
		30	29	
		31	53	
		33	32	
		33	45	
		34	35	
		36	37	
		38	39	
		42	40	
		47	41	
		43	44	
		46	51	
		48	49	
		50	52	
		54	55	
		56	57	
		59	58	
		60	61	
		62	63	
		64	65	
		66	67	
		68	69	
		70	71	
		72	56	

5. CONCLUSIONS

In this paper, the optimal placement of PMUs is performed by a new method based on the branch placement of the PMUs. The proposed method makes network totally observable by determining the optimal location of the PMUs. Since the proposed method adopts single channel PMUs, the installation cost is reduced while the network observability is maintained. The PMU placement on the real 400 kV network of Iran is also performed after being applied to the IEEE standard networks. Study of the obtained results shows that branch placement method could increase observability security index in different possible conditions of outages.

Table 7. Comparison of the installation costs of two placement methods

Method / Network	Bus placement	Branch placement
	Cost($)	Cost($)
24 buses	19000	18000
30 buses	24500	22500
400 kV of Iran	58500	55500

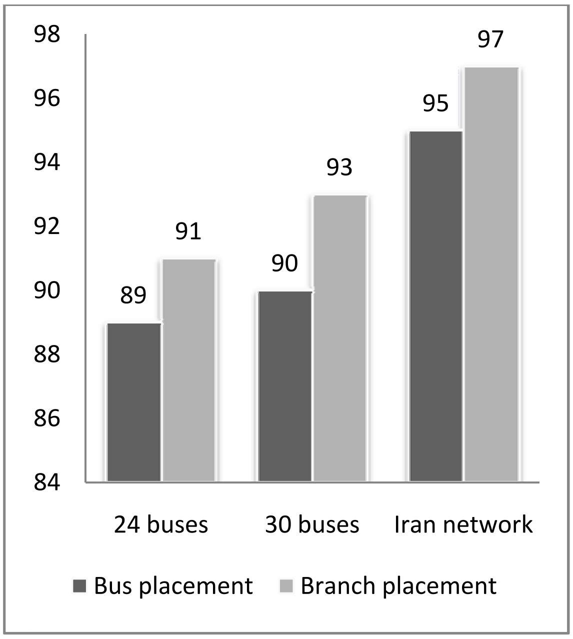

Fig. 3. Chart of mean observability security index

Table 8. Observability security index for the studied networks when a PMU or a line is eliminated

Method / Network	Bus placement: PMU and line outage	r	E= r/N	Branch placement: PMU and line outage	r	E= r/N
24 buses	PMU on bus 10	21	0.87	PMU in line 4-9	22	0.91
	PMU on bus 16	20	0.83	PMU in line 19-20	22	0.91
	PMU on bus 23	22	0.91	PMU in line 14-16	22	0.91
	Line 1-2	22	0.91	Line 21-22	22	0.91
	Line 7-8	23	0.96	Line 4-9	22	0.91
	Average		**0.89**	**Average**		**0.91**
30 buses	PMU on bus 10	26	0.86	PMU in line 6-8	28	0.93
	PMU on bus 12	25	0.83	PMU in line 23-24	28	0.93
	PMU on Bus 3	28	0.93	PMU in line 12-13	28	0.93
	Line12-13	29	0.96	Line 2-1	28	0.93
	Line 9-11	28	0.93	Line 5-7	28	0.93
	Average		**0.9**	**Average**		**0.93**
400 kV of Iran	PMU on bus 18	69	0.95	PMU in line 19-26	70	0.97
	PMU on bus 33	67	0.93	PMU in line 33-45	71	0.98
	PMU on bus 56	70	0.97	PMU in line 66-67	70	0.97
	PMU on bus 69	68	0.94	PMU in line 42-40	70	0.97
	Line 64-65	71	0.98	Line 19-26	70	0.97
	Line 3-4	71	0.98	Line 31-53	70	0.97
	Line 9-12	71	0.98	Line 56-57	70	0.97
	Average		**0.95**	**Average**		**0.97**

Table 9. Comparison between the proposed method and the method presented in [14]

Method / Network	The Method of [14]: Number of PMU	Cost $	The Proposed Method: Number of PMU	Cost $
24 buses	12	18000	12	18000
30 buses	15	22500	15	22500
400 kV of Iran	39	58500	37	55500

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