Journal of Operation and Automation in Power Engineering

Vol. 14, No. 4, Dec. 2026, Pages: 241-248

http://joape.uma.ac.ir

*Research Paper*

# Enhancing State Estimation Accuracy in Distribution Networks: An Optimized Algorithm for Strategic Meter Placement

*Ali Farahani Shahvarooghi* [1], *Ehsan Heydarian-Forushani* [1,*], *Mehdi Zeraati* [2], and *Saeed Hasanzadeh* [1]

[1]*Department of Electrical and Computer Engineering, Qom University of Technology, Qom, Iran.*
[2]*Power Systems Operation and Planning Research Department, Niroo Research Institute (NRI), Shahrak Ghods, Tehran, Iran.*

***Abstract***— *Accurate state estimation is crucial for the effective control and management of power grids, as it provides a comprehensive understanding of voltage magnitude and phase angle at network buses. Incorrect estimations may lead to damaging decisions and network collapse. This paper addresses the significance of precise state estimation in distribution networks and proposes an efficient algorithm for optimal measurement device allocation, aiming to minimize estimation errors. The algorithm considers both investment and technical constraints, utilizing an optimal alternative current (AC) power flow model that eliminates the need for exact values of active and reactive load demands. The proposed method identifies optimal locations for installing a specific number of phasor measurement units (PMUs) across the network. The application of the algorithm to 33-bus and 69-bus test systems demonstrates its effectiveness in enhancing state estimation accuracy. Results reveal that optimizing the number and location of measurement devices significantly improves outcomes. A comparative analysis with the conventional weighted least squares (WLS) algorithm underscores the applicability of the proposed model, particularly in distribution networks with limited measurement devices. The proposed method formulates optimal meter placement problems in distribution networks based on an optimal power flow model, which has a superior performance in both accuracy and convergence without needing to exact nodal demands for state estimation. This research contributes to the advancement of state estimation procedures, offering a practical approach to enhance accuracy and reliability in power grid management.*

***Keywords***—Alternative current optimal power flow, distribution network, meter placement, state estimation.

## NOMENCLATURE

**Indices**

| | |
|---|---|
| $l$ | Index of branches |
| $m/k$ | Index of network buses |

**Abbreviations**

| | |
|---|---|
| ACOPF | Alternative Current Optimal Power Flow |
| DG | Distributed Generation |
| DMS | Distribution Management System |
| DSM | Demand-Side Management |
| DSSE | Distribution System State Estimation |
| FZIB | Fully Zero Injection Buses |
| MILP | Mixed Integer Linear Programming |
| PMU | Phasor Measurement Unit |
| PZIB | Partially Zero Injection Buses |
| RER | Renewable Energy Resources |
| WLS | Weighted Least Squares |

**Variables and Parameters**

| | |
|---|---|
| $\Delta^V, \Delta^\theta$ | Maximum error of measurement device |
| $\Delta_P^{load}$ | Maximum allowable change of active powers compared to power flow values |
| $\Delta_Q^{load}$ | Maximum allowable change of reactive powers compared to power flow values |
| $\hat{\theta}_k$ | Estimated voltage phase angle of bus $k$ |
| $\hat{P}_k^{load}, \hat{Q}_k^{load}$ | Estimated active/reactive load demand at bus $k$ |
| $\hat{V}_k$ | Estimated voltage magnitude of bus $k$ |
| $\text{PMU}_k$ | Binary variable indicating presence of PMU at bus $k$ |
| $\theta^{max}, \theta^{min}$ | Maximum/Minimum allowed voltage phase angle |
| $\theta_k^{pf}$ | Voltage phase angle at bus $k$ from AC power flow |
| $A_{km}$ | Matrix associated with the connectivity of buses |
| $B_{km}^{Line}$ | Susceptance of branch between bus $k$ and bus $m$ |
| $G_{km}^{Line}$ | Conductance of branch between bus $k$ and bus $m$ |
| $I_{km}^{Line}$ | Current in branch between bus $k$ and bus $m$ |
| $I_{km}^{max}$ | Maximum allowed current of branch between bus $k$ and bus $m$ |
| $No$ | Maximum number of available meters |
| $P_k^{Gen}, Q_k^{Gen}$ | Active/Reactive power generated at bus $k$ |
| $P_k^{load}, Q_k^{load}$ | Active/Reactive load demand at bus $k$ from AC power flow |
| $P_{km}^{Line}, Q_{km}^{Line}$ | Active/Reactive power flow in branch between bus $k$ and bus $m$ |
| $V^{max}, V^{min}$ | Maximum/Minimum allowed voltage magnitude |
| $V_k^{pf}$ | Voltage magnitude at bus $k$ from AC power flow |
| $Z$ | Objective function value |

Received: 18 Jun. 2024
Revised: 21 Jan. 2025
Accepted: 02 Feb. 2025
*Corresponding author:
E-mail: heydarian@qut.ac.ir (E. Heydarian-Forushani)
DOI: 10.22098/joape.2025.15292.2163



## 1. INTRODUCTION

Electrical distribution networks are recognized as highly intricate engineering systems characterized by their extensive infrastructure and substantial equipment inventory. While the primary function of distribution networks is to reliably and economically supply electricity to consumers, they face a myriad of faults and disturbances that pose risks to their performance [1]. The emergence of smart grids, incorporating distributed generation (DG), renewable energy resources (RER), and demand-side management (DSM), has altered the traditional unidirectional power flow in distribution feeders to a bidirectional mode. While this transformation brings economic and environmental benefits, it also presents significant technical challenges requiring substantial operational changes in electrical distribution networks [2]. In light of these developments, ensuring the accurate and real-time monitoring of distribution networks becomes imperative for effective control and security maintenance. The monitoring process involves collecting data from the distribution network through installed measuring equipment, which is transmitted to the control center via telecommunication links. The subsequent data processing is carried out by the distribution management system (DMS), with the distribution system state estimator serving as a fundamental component in the security evaluation of power distribution networks. As the primary module within the DMS, the state estimator plays a pivotal role in refining input data for other modules [3]. The power system state variables, specifically the voltage magnitude and phase angle of network buses, offer a holistic understanding of the network. Conversely, inaccurate estimation of these variables can lead to erroneous decisions, potentially resulting in severe damages or network collapse.

To address this, employing appropriate distribution system state estimation (DSSE) algorithms is crucial for accurately assessing the operational status of the system. However, the efficacy of state estimation relies on an adequate number of strategically placed measuring devices throughout the power distribution network [4]. Establishing an optimal measurement infrastructure becomes paramount to supplying the requisite real-time data for DSSE algorithms to generate precise results. Given the numerous substations and feeders in distribution networks, it is economically unfeasible to install measuring equipment in every substation. Hence, determining the optimal number and location of measuring devices becomes a critical technical and economic consideration [5].

In [6], the allocation of measurement devices in distribution networks is tackled with a dual focus on reducing annual energy losses and enhancing the accuracy of the state estimator. These objectives are addressed through a multi-objective formulation solved by the Biased Random-Key Genetic Algorithm. In [7], a method is proposed utilizing dominance and decomposition techniques, framing the optimal placement of meters as a constrained multi-objective optimization problem that considers the relative error percentage of average voltage magnitude and phase angle, along with cost considerations. Authors in [8] present two meter placement methods based on system parameters and power flow equations to determine optimal installation locations, aiming to minimize voltage residuals at selected reference states. The authors in [6–8] propose a multi-objective optimization, which necessarily does not result in the best DSSE accuracy. In [9], a $\mu$PMU placement technique is introduced, emphasizing fault detection in distribution networks. Methods like partially zero injection buses (PZIB) and fully zero injection buses (FZIB) [10] are employed for optimal $\mu$PMU placement, formulated as a binary integer linear programming problem. [11] presents a topology estimation approach for low voltage distribution networks using smart meters and energy meters of customers. A sensor placement strategy is proposed in [12] to capture voltage magnitude fluctuations in distribution networks, which detect all possible voltage limit violations under multiple switching configurations. However, [9–12] do not address the accuracy of state estimation, potentially leading to suboptimal results for DSSE.

A multi-objective evolutionary algorithm for meter placement in active distribution networks is presented in [13], aiming to minimize both total cost and state estimation error. [14] explores the impact of smart meter placement on DSSE algorithms, suggesting installing smart meters on buses with the highest energy consumption. Applying only smart meters could not deliver high accuracy. In [15], assuming a radial distribution network, measuring devices are installed at load buses and slack bus, providing voltage magnitude and active/reactive power flow data. The proposed method is heuristic and it is not necessarily the optimal solution. [16] utilizes Cuckoo search optimization for optimal general meter and PMU placement, while [17] minimizes DSSE error using mixed-integer linear programming (MILP). [18] investigates real-time monitoring requirements for advanced volt-var control, defining a meter placement problem for CVR monitoring, yet not considering voltage phase angle estimation and $\mu$PMU placement modeling, potentially leading to inaccurate branch current estimation and network loss [19].

Similarly, [20] proposes an evolutionary algorithm for meter placement in active distribution networks, considering investment cost and relative error of voltage magnitude and phase angle. [21] employs a multi-objective optimization model to balance low investment costs with acceptable state estimation performance. [22] introduces a multi-objective function for PMU placement, considering all state estimation error components with adaptive decision coefficients. In [23], a multi-objective evolutionary algorithm is used to minimize investment cost and enhance DSSE accuracy, implemented through particle swarm optimization and validated with Monte Carlo simulation. [24] incorporates voltage and power flow measurement placement to improve DSSE accuracy, formulated as a mixed-integer semi-definite programming model. The authors in [25] propose an optimal meter placement strategy considering monitoring uncertainty and related economic issues. This algorithm is based on three modules: state estimation, optimal placement, and voltage control. In [20–25], DSSE problems are formulated using the weighted least squares (WLS) minimization method, addressing the challenge of ill-conditioning in the gain matrix. However, the computational complexity and stability of results remain concerns.

This paper focuses on improving DSSE accuracy, considering meter placement as a multi-objective optimization problem, where various factors such as budget, operator preferences, and technical aspects are treated as constraints. It is noteworthy that the increased use of $\mu$PMUs in recent years, due to their ability to measure voltage magnitude and phase angle with high accuracy in microseconds, prompts this paper to explore the optimal allocation of these devices in distribution networks for accurate branch current estimation and minimizing power loss in distribution feeders. In a distribution network, the state estimator aims to provide the best estimate of state variables despite errors in measured values. These measured values are categorized into actual measurements (voltage, current, and power flow values) and pseudo-measurements (estimated or predicted values of network loads with high variance). Limited real-time measurements, usually confined to the main substation, make online monitoring of feeder measurements challenging. Consequently, DSSE for power distribution networks often relies on pseudo-measurements. Designing a DSSE algorithm with high accuracy in the presence of a limited number of real-time measurements is crucial. This study investigates determining the optimal combination of meters for monitoring a distribution network with a restricted number of real-time measurements. The key contributions of this paper are outlined as follows:

- Formulate the problem of optimal placement of $\mu$PMUs in distribution networks to enhance state estimation accuracy, utilizing the AC optimal power flow (ACOPF).
- Incorporate pseudo-measurements, such as historical or forecasted load data, into the DSSE problem to achieve the desired state estimation accuracy, even when faced with a restricted number of real-time measurements.
- Enhance both the convergence and accuracy of the output,

surpassing conventional methods based on WLS.

The subsequent sections of this paper are organized as follows. Section 2 introduces the model for optimal placement of $\mu$PMUs in distribution networks, with the objective of enhancing state estimation accuracy through the ACOPF model. Section 3 is dedicated to presenting simulation results and assessing the effectiveness of the proposed algorithm in diverse case studies. Concluding remarks are provided in Section 4.

## 2. PROPOSED MODEL

In this section, the conventional WLS algorithm for state estimation in power systems is briefly introduced. Then, the proposed algorithm is presented in two separate subsections: objective function and problem formulation. Finally, the overall solution procedure is explained in Subsection 2.4.

### 2.1. WLS algorithm

The weighted least squares (WLS) is one of the most commonly used algorithms to solve the state estimation problem in power systems. This method usually minimizes the following objective function which provides the maximum likelihood estimation [19]:

$$MinJ(x) = [z - h(x)]^T R_z^{-1}[z - h(x)] \tag{1}$$

For a given network configuration, a nonlinear equation expressing the relationship between measurements and system states can be written as:

$$z = h(x) + e_z \tag{2}$$

where $x$ and $z$ are $(n \times 1)$ state vector and $(m \times 1)$ measurement vector, respectively in which $n$ and $m$ are number of network buses and measurement, respectively. h(x) is the vector of $m$ non-linear functions linking the m measured variables to the $n$ estimated state. If $\sigma_{zi}^2$ is assumed to be the variance of $i^{th}$ measurement, $e_z \sim N(O,\ R_z)$ is a Gaussian noise vector with a mean of zero and a measurement error covariance matrix $\mathrm{R_z}=\mathrm{diag}\{\sigma_{\mathrm{z1}}^2,\ \sigma_{\mathrm{z2}}^2,\ \sigma_{\mathrm{zm}}^2\}$.

The estimated states' vector $\hat{\mathrm{x}}$ that minimizes J(x) is obtained from the following condition [19]:

$$\frac{\partial J(x)}{\partial x}|_{x=\hat{x}} = -H^T(\hat{x})R_z^{-1}[z - H(\hat{x})] = 0 \tag{3}$$

The estimated vector can be calculated by Jacobian matrix $H = \frac{\partial h(x)}{\partial x}|_{x=\hat{x}}$ as:

$$\hat{x} = G^{-1}H^T R_z^{-1} z \tag{4}$$

The matrix $G$ is called the Gain matrix and formulated as $\mathrm{G} = \mathrm{H}^T R_z^{-1} H$. If the estimation error is defined as $|x - \hat{x}|$, then the optimal meters placement problem will lead to the selection of the vector $z$ by installing the limited number of measurements in the network buses in such a way that $|x - \hat{x}|$ remains minimum.

However, provided that the network is fully observable, the matrix G will be symmetric, sparse, and positive definite. The network is observable when the size of the independent measurement vector is more than the number of states in the system. Therefore, as the real-time meters are very limited in distribution networks, they are not sufficient for making the system fully observable. For this reason, WLS method is not suitable for DSSE in networks with low observability.

### 2.2. Objective function

In this section, the proposed model for the optimal placement of $\mu$PMUs in distribution networks is discussed based on optimal AC power flow equations. Since the main goal of this paper is to improve the accuracy of the DSSE algorithm, the objective function of the optimization problem is defined as follows:

$$Z = \sum_{k=1}^{n} \left\{ \left| \frac{\hat{V}_k - V_k^{pf}}{V_k^{pf}} \right| + \left| \hat{\theta}_k - \theta_k^{pf} \right| \right\} \tag{5}$$

In Eq. (5), $Z$ represents the state estimation error in terms of network state variables (voltage magnitude and phase angle). $\hat{V}_k$ and $\hat{\theta}_k$ are the estimated state variables resulting from the DSSE algorithm, while the $V_k^{pf}$ and $\theta_k^{pf}$ are respectively, magnitude and phase angle of the bus voltages which are obtained from the real-time measurements. As observed, the relative error between the estimated values and the outcomes of AC power flow values (at a certain snapshot) is considered as the estimation error. The first term of the objective function ($|\frac{\hat{V}_k - V_k^{pf}}{V_k^{pf}}|$) is the normalized estimation error of voltage magnitudes. Similarly, the second term ($\left|\hat{\theta}_k - \theta_k^{pf}\right|$) is the total estimation error of voltage phase angles.

It should be noted that due to the lack of real measurement data for test networks, the power flow results in a certain state of the network have been used as the output of the measurements. Moreover, to avoid numerical problems, the second term of the objective function is adopted to calculate the relative phase angle error. Because the phase angles are usually very small in distribution networks. This helps two terms of objective function remain at the same level and hence, the optimization algorithm considers scalable weight for both terms.

### 2.3. Problem formulation

Eq. (6) to Eq. (16) show the mathematical formulation of the proposed $\mu$PMU placement problem based on ACOPF.

$$P_k^{Gen} - \hat{P}_k^{load} = \sum_{m=1}^{n} A_{km} P_{km}^{Line} \tag{6}$$

$$Q_k^{Gen} - \hat{Q}_k^{load} = \sum_{m=1}^{n} A_{km} Q_{km}^{Line} \tag{7}$$

$$P_{km}^{Line} = \hat{V}_k^2 G_{km}^{Line} - \hat{V}_k \hat{V}_m G_{km}^{Line} cos(\hat{\theta}_k - \hat{\theta}_m) - \hat{V}_k \hat{V}_m B_{km}^{Line} sin(\hat{\theta}_k - \hat{\theta}_m) \tag{8}$$

$$Q_{km}^{Line} = -\hat{V}_k^2 B_{km}^{Line} + \hat{V}_k \hat{V}_m B_{km}^{Line} cos(\hat{\theta}_k - \hat{\theta}_m) - \hat{V}_k \hat{V}_m G_{km}^{Line} sin(\hat{\theta}_k - \hat{\theta}_m) \tag{9}$$

$$I_{km}^{Line} = \sqrt{\left[(G_{km}^{Line})^2 + (B_{km}^{Line})^2\right]\left[\hat{V}_k^2 + \hat{V}_m^2 - 2\hat{V}_k \hat{V}_m cos(\hat{\theta}_k - \hat{\theta}_m)\right]} \tag{10}$$

$$-I_{km}^{\max} \le I_{km}^{Line} \le I_{km}^{\max} \tag{11}$$

$$(1 - \Delta_P^{load}) P_k^{load} \le \hat{P}_k^{load} \le (1 + \Delta_P^{load}) P_k^{load} \tag{12}$$

$$(1 - \Delta_Q^{load}) Q_k^{load} \le \hat{Q}_k^{load} \le (1 + \Delta_Q^{load}) Q_k^{load} \tag{13}$$

$$V^{min} + [(1-\Delta^V)V_k^{pf} - V^{min}]PMU_k \leq \hat{V} \leq V^{max} + [(1+\Delta^V)V_k^{pf} - V^{max}]PMU_k \quad (14)$$

$$\theta^{min} + [(1-\Delta^\theta)\theta_k^{pf} - \theta^{min}]PMU_k \leq \hat{\theta} \leq \theta^{max} + [(1+\Delta^\theta)\theta_k^{pf} - \theta^{max}]PMU_k \quad (15)$$

$$\sum_{k=1}^{n} PMU_k \leq No \quad (16)$$

Eqs. (6) and (7) represent the active and reactive power balance at each bus of the network, respectively. $A_{km}$ is the element $(k, m)$ of the network adjacency matrix such that if there is a branch between buses $k$ and $m$, $A_{km} = 1$ and otherwise $A_{km} = 0$. $P_k^{Gen}$ and $Q_k^{Gen}$ are the active and reactive power generation at each bus, respectively. Note that, these values are equal to zero for all buses except the first bus at the beginning point of the feeder. However, if there is distributed generation in the network, it can be easily modeled through these variables.

The active and reactive power consumption at each bus are modeled through $\hat{P}_k^{load}$ and $\hat{\theta}_k^{load}$, respectively. As it can be seen, these variables are considered as estimated parameters so that in cases where the load data in the distribution networks are not precisely known, it can help to improve the estimation procedure. More details are given in the description of Eqs. (8) and (9) in the following.

Eqs. (8) and (9) express the active and reactive power flows through the branches of the network which are dependent on the estimated voltage magnitude and phase angle as well as the branch's parameters. It is noticed that, $G_{km}^{Line}$ and $B_{km}^{Line}$ are the elements belonging to the network conductance and susceptance matrix. The current passing through the branches of the network is also obtained from Eq. (10). Moreover, the thermal capacity limitations of the lines are also taken into account in Eq. (11).

Eqs. (12) and (13) model the minimum and maximum limits of the estimated active and reactive power of loads. Since the exact amounts of loads are usually not known in the database of power distribution companies to carry out power flow calculations, so these values are known as pseudo-measurements and are determined using historical data. Since the accuracy of pseudo-measurements is often low and it can have a significant impact on the estimation results, therefore, an acceptable range for active and reactive power of the loads at each bus is intended here. This could help the optimization problem to adjust the values of these quantities so that the best estimation is achieved according to the actual measurements made in the network. On this basis, flexibility of the formulation will be increased to achieve better results. $\Delta_P^{load}$ and $\Delta_Q^{load}$ are the permissible ranges for the changes of estimated active and reactive power of loads according to their historical data.

Eq. (14) limits the voltage of network buses. As it can be seen, this equation specifies the voltage range in two ways depending on the existence of $\mu$PMU in the related bus. If there is no measuring device on a bus ($PMU_k = 0$), the estimated voltage must fall within the permissible voltage range (e.g. 0.9-1.0 pu). On the other hand, if the measuring device is placed in a bus ($PMU_k = 1$), the bus voltage is set to the actual measured value which is obtained from AC power flow considering a small deviation equal to the measurement error ($\Delta^V$). Similarly, Eq. (15) also applies the permissible changes of the voltage phase angle of the network buses in the case of the existence/absence of $\mu$PMU in the corresponding bus.

Finally, $PMU_k$ is a binary decision variable vector that is considered equal to 1 if bus k is selected for meter installation and vice versa. Consequently, Eq. (16) is used to determine the maximum number of available meters that should be installed in the network subject to investment constraints. Note that it is assumed a $\mu$PMU is installed on bus No.1 to measure the magnitude and phase angle of the voltage. It means that if the number of $\mu$PMUs that can be installed in the system is set to one, the allocation algorithm will place a meter in the first bus, automatically. In addition, the magnitude and phase angle of bus No. 1 are set to 1 per unit and zero, respectively.

### 2.4. Solution procedure

The overall solution procedure is depicted in Fig. 1. At the first step, network data (including adjacency matrix, thermal capacity and impedance of power lines), load and generation data (based on historical profiles) and fixes parameters (such as maximum and minimum limits of variables) are required inputs to the model. Next, the optimal meter placement problem is formulated using the proposed algorithm based on ACOPF. It should be noted that a power flow analysis should be performed to determine a specific snapshot of the network. The power flow results are needed to utilize as the real measurements on studied test networks. At third step, the estimated magnitudes and phase angels of nodal voltages will be the output results to calculate the DSSE error as the objective function. Finally, the binary vector $PMU_k$ shows the obtained optimal location of meters to maximize the DSSE accuracy using a limited number of $\mu$PMUs.

## 3. Simulation Results

In order to evaluate the proposed meter placement algorithm, different scenarios are defined on two well-known test systems in this section and the simulation results are presented in the form of various figures and tables. Fig. 2 shows the single line diagrams of IEEE 33 and 69 bus test networks. The data related to branch parameters and load values is available in [26].

As discussed before, due to the unavailability of real measurements on the test networks, the results of AC power flow at a certain snapshot are used as the values of real measurements in order to evaluate the accuracy of state estimation results. Due to the fact that the meter used in this study is $\mu$PMU, the voltage magnitude and phase angle of the buses on which $\mu$PMU is installed is assumed as real measurements. The power flow results (voltage magnitude and phase angle of network buses) for IEEE 33 and 69 bus test systems are given in Tables 1 and 2, respectively.

In simulation studies, all the loads have an uncertainty range of $\pm 40\%$ ($\Delta_P^{load} = \Delta_Q^{load} = 0.4$). This means that in Eqs. (12) and (13), $\hat{P}_k^{load}$ and $\hat{\theta}_k^{load}$ are set within the range of 0.6 and 1.4 times the approximate values considered as the primary pseudo-measurements. This tolerance is reasonable for the worst errors in load forecasts. Furthermore, the maximum measurement error of voltage magnitude ($\Delta^V$) and phase angle ($\Delta^\theta$) by $\mu$PMU is considered 0.3%. This error percentage is normal for industrial $\mu$PMUs [17]. The minimum and maximum allowed magnitude for nodal voltages ($V^{min}/V^{max}$) is selected as 0.9 and 1.1 p.u., respectively which are common operation limits in most distribution network companies. $\theta^{min}$ and $\theta^{max}$ are set to -1 and +1 according to the nominal minimum and maximum for operational voltage phase angles.

It must be noted that the simulations were performed in GAMS software using a PC with an Intel Core i7 2.80 GHz processor and 4.00 GB RAM. To check the performance of the proposed method, the following three scenarios are defined and the obtained results are compared from different aspects:

*A) Evaluating the effect of the number of meters on DSSE accuracy*

Firstly, the effect of $\mu$PMU's number on the objective function is investigated. Due to the limited budget of distribution companies, there are usually a limited number of measuring devices that should be installed in the best places throughout the network. In this scenario, by increasing the number of $\mu$PMUs and solving the optimization problem, the best candidate buses for $\mu$PMU

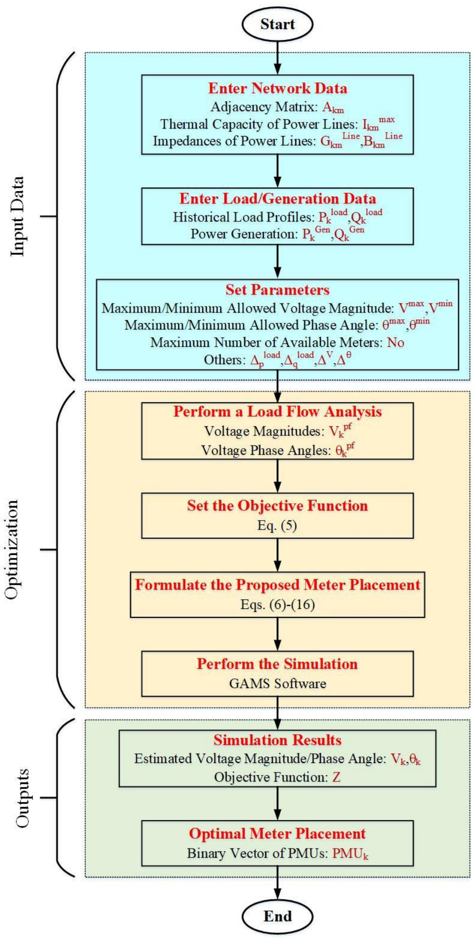


Fig. 1. Solution procedure.

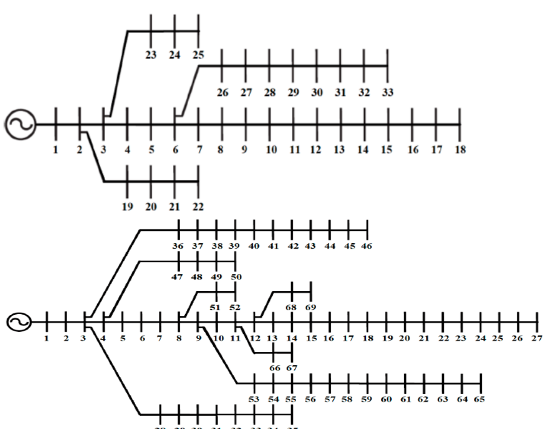


Fig. 2. IEEE 33-bus and 69-bus test systems [26].

installation are specified and the effect of the number of meters on the accuracy of the system state estimation is reported.

*B) Comparing the estimated values of voltage magnitude and phase angle with the actual measured values in order to check the accuracy of state estimation on the test networks*

In this section, for some cases with a certain number of PMUs, the estimated voltage magnitude and phase angle of the network buses are compared with their actual values considering the optimal installation of $\mu$PMUs.

Table 1. Power flow results for IEEE 33-bus test system.

| Bus | Voltage magnitude (pu) | Voltage phase angle (Degree) | Bus | Voltage magnitude (pu) | Voltage phase angle (Degree) |
|---|---|---|---|---|---|
| 1 | 1.000 | 0.000 | 18 | 0.913 | -0.495 |
| 2 | 0.997 | 0.014 | 19 | 0.997 | -0.004 |
| 3 | 0.983 | 0.096 | 20 | 0.993 | -0.063 |
| 4 | 0.975 | 0.162 | 21 | 0.992 | -0.083 |
| 5 | 0.968 | 0.228 | 22 | 0.992 | -0.103 |
| 6 | 0.950 | 0.134 | 23 | 0.979 | 0.065 |
| 7 | 0.946 | -0.096 | 24 | 0.973 | -0.024 |
| 8 | 0.941 | -0.060 | 25 | 0.969 | -0.067 |
| 9 | 0.935 | -0.133 | 26 | 0.948 | 0.173 |
| 10 | 0.929 | -0.196 | 27 | 0.945 | 0.229 |
| 11 | 0.928 | -0.189 | 28 | 0.934 | 0.312 |
| 12 | 0.927 | -0.177 | 29 | 0.948 | 0.390 |
| 13 | 0.921 | -0.269 | 30 | 0.922 | 0.496 |
| 14 | 0.919 | -0.347 | 31 | 0.918 | 0.411 |
| 15 | 0.917 | -0.385 | 32 | 0.917 | 0.388 |
| 16 | 0.921 | -0.408 | 33 | 0.917 | 0.380 |
| 17 | 0.914 | -0.485 | | | |

*C) Comparison with the WLS-based state estimation method*

Finally, to evaluate the performance of the proposed formulation in comparison with other common methods of state estimation and meter placement, the obtained results are compared with the conventional WLS method.

Table 2. Power flow results for IEEE 69-bus test system.

| Bus | Voltage magnitude (pu) | Voltage phase angle (Degree) | Bus | Voltage magnitude (pu) | Voltage phase angle (Degree) |
|---|---|---|---|---|---|
| 1 | 1.000 | 0.000 | 36 | 1.000 | -0.003 |
| 2 | 1.000 | -0.001 | 37 | 1.000 | -0.009 |
| 3 | 1.000 | -0.002 | 38 | 1.000 | -0.011 |
| 4 | 1.000 | -0.006 | 39 | 1.000 | -0.012 |
| 5 | 0.995 | 0.023 | 40 | 1.000 | -0.013 |
| 6 | 0.990 | 0.049 | 41 | 0.999 | -0.024 |
| 7 | 0.981 | 0.121 | 42 | 0.999 | -0.026 |
| 8 | 0.979 | 0.138 | 43 | 0.999 | -0.029 |
| 9 | 0.977 | 0.147 | 44 | 0.998 | -0.030 |
| 10 | 0.972 | 0.232 | 45 | 0.998 | -0.031 |
| 11 | 0.971 | 0.251 | 46 | 0.998 | -0.031 |
| 12 | 0.968 | 0.304 | 47 | 1.000 | -0.042 |
| 13 | 0.965 | 0.350 | 48 | 0.999 | -0.053 |
| 14 | 0.962 | 0.397 | 49 | 0.995 | -0.192 |
| 15 | 0.960 | 0.400 | 50 | 0.994 | -0.211 |
| 16 | 0.959 | 0.451 | 51 | 0.979 | 0.139 |
| 17 | 0.958 | 0.466 | 52 | 0.979 | 0.139 |
| 18 | 0.958 | 0.466 | 53 | 0.975 | 0.169 |
| 19 | 0.958 | 0.473 | 54 | 0.971 | 0.195 |
| 20 | 0.957 | 0.480 | 55 | 0.967 | 0.230 |
| 21 | 0.957 | 0.489 | 56 | 0.956 | 0.410 |
| 22 | 0.957 | 0.489 | 57 | 0.945 | 0.590 |
| 23 | 0.957 | 0.491 | 58 | 0.934 | 0.770 |
| 24 | 0.957 | 0.493 | 59 | 0.925 | 0.945 |
| 25 | 0.956 | 0.495 | 60 | 0.918 | 0.982 |
| 26 | 0.956 | 0.497 | 61 | 0.912 | 1.119 |
| 27 | 0.956 | 0.498 | 62 | 0.912 | 1.122 |
| 28 | 1.000 | -0.003 | 63 | 0.911 | 1.132 |
| 29 | 1.000 | -0.005 | 64 | 0.910 | 1.143 |
| 30 | 1.000 | -0.003 | 65 | 0.909 | 1.148 |
| 31 | 1.000 | -0.002 | 66 | 0.971 | 0.252 |
| 32 | 0.999 | 0.001 | 67 | 0.971 | 0.252 |
| 33 | 0.999 | 0.003 | 68 | 0.968 | 0.310 |
| 34 | 0.999 | 0.009 | 69 | 0.968 | 0.310 |
| 35 | 0.999 | 0.010 | | | |

Table 3. Optimal location of $\mu$PMUs for IEEE 33-bus test system.

| Number of $\mu$PMUs | DSSE Error | Installation Location |
|---|---|---|
| 1 | 0.559 | 1 |
| 2 | 0.403 | 1,3 |
| 3 | 0.310 | 1,3,33 |
| 4 | 0.254 | 1,3,14,19 |
| 5 | 0.183 | 1,3,4,13,19 |
| 6 | 0.127 | 1,2,3,19,21,23 |
| 7 | 0.084 | 1,2,3,5,17,25,27 |
| 8 | 0.079 | 1,5,11,15,16,17,19,20 |
| 9 | 0.059 | 1,2,3,13,16,17,19,20,25 |
| 10 | 0.014 | 1,2,4,7,9,17,20,24,27,29 |

### 3.1. Evaluating the effect of the number of meters on DSSE accuracy

In this scenario, the number of $\mu$PMUs is increased (according to Eq. (16)) step by step and the proposed algorithm determines optimal location of $\mu$PMUs to minimize the state estimation error. In fact, this algorithm can suggest optimal locations of the meters

Table 4. Optimal location of $\mu$PMUs for IEEE 69-bus test system.

| Number of $\mu$PMUs | DSEE error | Installation location |
|---|---|---|
| 1 | 0.679 | 1 |
| 2 | 0.580 | 1,45 |
| 3 | 0.451 | 1,28,43 |
| 4 | 0.365 | 1,3,28,36 |
| 5 | 0.170 | 1,3,7,35,51 |
| 6 | 0.149 | 1,3,28,36,39,51 |
| 7 | 0.119 | 1,3,8,28,36,39,51 |
| 8 | 0.114 | 1,3,4,28,35,36,51,66 |
| 9 | 0.109 | 1,3,4,28,35,36,39,51,66 |
| 10 | 0.079 | 1,3,4,8,28,35,36,40,48,51 |
| 15 | 0.059 | 1,3,4,11,12,28,35,36,39,40,46,48,50,51,69 |
| 20 | 0.025 | 1,3,4,11,12,14,16,17,18,20,21,26,27,28,35,36,39,46,51,67 |
| 25 | 0.016 | 1,3,4,9,11,12,13,18,21,22,26,27,28,33,35,36,39,43,46,48,50,55,62,65,67 |

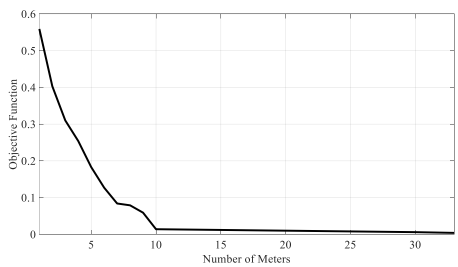


Fig. 3. Objective function values for different number of meters (33-bus test system).

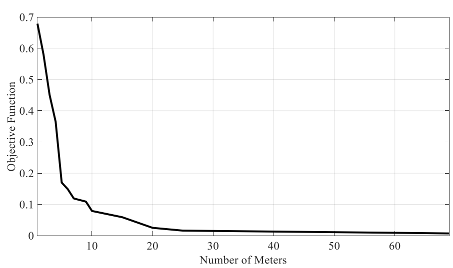


Fig. 4. Objective function values for different number of meters (69-bus test system).

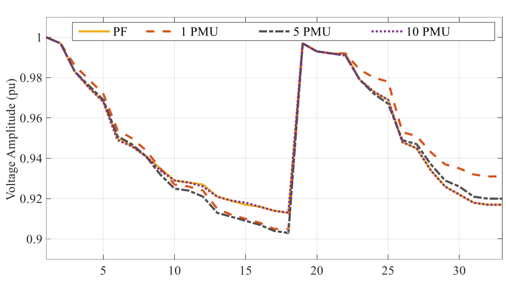


Fig. 5. Estimated voltage magnitude for different number of meters (33-bus test system).

according to the available number of meters with the aim of decreasing the state estimation error.

According to the previous studies and based on the experimental results, usually the required number of meters to achieve the desired DSSE result is equal to one-third of the number of network buses [19]. For example, the required number of meters is about 11 for the 33-bus test network. In addition, due to the limited budget and network operator's unwillingness to use several meters

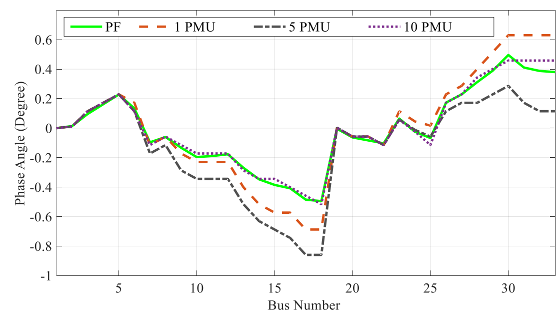


Fig. 6. Estimated voltage phase angle for different numbers of meters (33-bus test system).

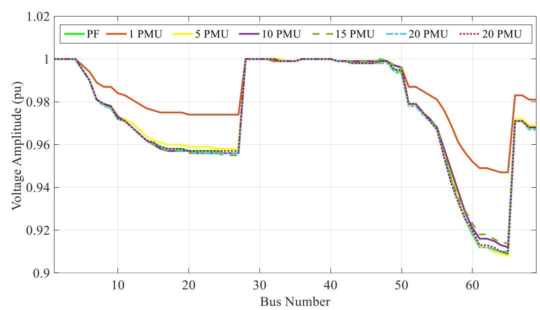


Fig. 7. Estimated voltage magnitude for different number of meters (69-bus test system).

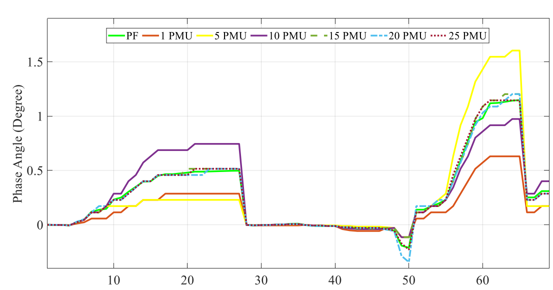


Fig. 8. Estimated voltage phase angle for different number of meters (69-bus test system).

due to their high costs, the maximum value of one-third seems reasonable. However, it is obvious that as the number of meters increases, the state estimation error will be decreased.

For the IEEE 33-bus test network, the number of $\mu$PMUs has

Table 5. Comparison of DSSE error between the proposed method and conventional WLS (33-bus test system).

| Method/Number of $\mu$PMUs | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Proposed method (%) | 1 | 0.72 | 0.55 | 0.45 | 0.32 |
| WLS (%) | 1 | 0.96 | 0.83 | 0.62 | 0.54 |

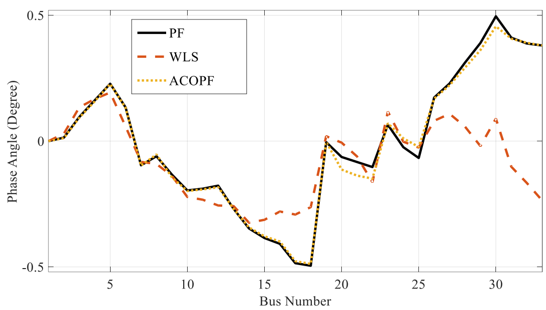


Fig. 9. Estimated voltage magnitude for two methods (33-bus test system).

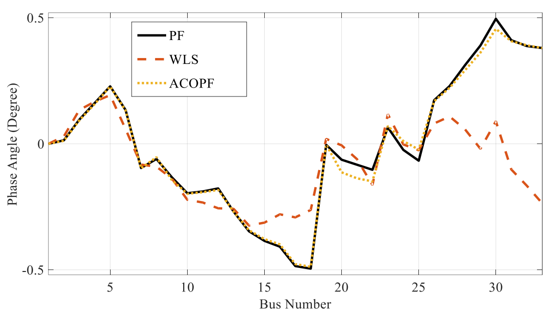


Fig. 10. Estimated voltage phase angle for two methods (33-bus test system).

been changed from 1 to 10. The simulation results including optimal location along with the state estimation error are summarized in Table 3. Moreover, the objective function values for different numbers of meters are illustrated in Fig. 3. As observed, the state estimation error does not change significantly after the placement of 10 $\mu$PMUs. So, it is not economically logical to increase the number of meters beyond 10. As expected, with the increase in the number of $\mu$PMUs, the state estimation error has also decreased. Considering the previous assumptions as well as changing the number of meters that can be installed in the network, the optimum number of $\mu$PMUs for the IEEE 69-bus test system has been investigated in the following. According to Fig. 4 and the obtained state estimation error in different cases, the DSSE error value is almost constant when the number of installed meters exceeds 25. For this reason, it is assumed that the number of installed $\mu$PMUs for the 69-bus test system could be 1 to 10, 15, 20 and 25. According to the above explanation, the optimal location of measuring devices as well as the state estimation error for each case have been reported in Table 4. Again, as the number of meters increases, the state estimation error decreases.

It is noteworthy that for both test systems, as the number of available meters increases, their optimal location is determined by the algorithm in such a way that more measurements are dedicated to sub-branches to increase the accuracy of DSSE.

### 3.2. Evaluating the accuracy of state estimation results

In this section, the accuracy of estimated voltage magnitude and phase angles by the proposed algorithm are evaluated on both test systems. Fig. 5 shows the voltage magnitude of the network buses in three different cases including 1, 5 and 10 $\mu$PMUs installed at optimal locations in IEEE 33-bus test system. In this figure, the AC power flow results are also displayed as real measurements. As it can be seen, by increasing the number of $\mu$PMUs, the estimated values get closer to the actual measurements. In fact, by increasing $\mu$PMU numbers and installing them in the optimum locations, the state estimation result gets closer to the real measurements. Fig. 6 also represents the estimated voltage phase angles of the network buses in the aforementioned situation and compares them with real measurements. The obtained results for the estimated voltage phase angles of the network buses reveal that accuracy of the DSSE algorithm is increased by installing more $\mu$PMUs.

An investigation on the impacts of changing the number of $\mu$PMUs with similar assumptions has also been performed for the IEEE 69-bus test system. According to Fig. 7, the accuracy of the state estimation algorithm will be improved and its outputs get closer to the actual measurements as a result of increasing the number of $\mu$PMUs. Fig. 8 also indicates the voltage phase angle values in the case of different numbers of $\mu$PMUs in the 69-bus test system. It can be concluded that the optimal number of $\mu$PMUs as well as their installation in the right places has a direct effect on accuracy of the state estimation algorithm.

Comparing Figs. 7 and 8, it can be seen that in general, the accuracy of the voltage magnitude estimation is more favorable than the voltage phase angle estimation. The main reason is that in distribution networks, due to the short length power lines, the voltage phase angles of adjacent buses are very close to each other, making it difficult to estimate these phase angles accurately from the limited available measurements. However, the overall performance of the state estimation algorithm is very acceptable by increasing the number of measurements to about a quarter of the number of network buses.

### 3.3. Result comparison with WLS method

In this section, the effectiveness of the proposed method is evaluated in comparison with the conventional WLS method that has been used in many previous works. For this purpose, the simulations are performed only on the IEEE 33-bus test system and the state estimation results are compared in both methods.

Table 5 compares the state estimation error of the proposed method with the WLS method in the cases with 1 to 5 $\mu$PMUs. As it can be seen, the percentage of DSSE error has decreased with increasing the number of $\mu$PMUs in both methods. However, the error reduction is much more noticeable than the WLS method for the proposed algorithm. In addition, with increasing the number of $\mu$PMUs, the error percentage has decreased more rapidly. Therefore, it can be concluded that the performance of the proposed method is better in comparison with the conventional WLS method.

To better understand the effectiveness of the proposed method compared to the WLS method, the state estimation results for IEEE 33-bus test system are presented by placing four $\mu$PMUs in the locations specified by the algorithms. The simulation results for the voltage magnitude and phase angles of the network buses are shown in Figs. 9 and 10. The simulation results prove that the performance of the proposed method in estimating the network condition is better than the conventional WLS method with the same number of $\mu$PMUs. It is noteworthy that this issue is also improved by increasing the number of meters.

The presented simulation results in this subsection provers that, especially in the case with low number of measurements, the conventional WLS-based DSSE algorithm fails to provide accurate state estimation in distribution networks. This problem has been reported in many studies in literature. The ACOPF-based DSSE algorithm proposed in this paper can overcome this challenge properly by providing better performance in low observable networks.

## 4. Conclusion

The optimal meter placement problem in distribution networks to improve the accuracy of the state estimation process is of great importance for the safe and economic operation of distribution networks. This paper presented an optimization-based model for $\mu$PMU placement in a distribution network to improve the accuracy of the DSSE algorithm. The effectiveness of the proposed algorithm was evaluated on two test networks through simulation in GAMS software. Moreover, the obtained results were compared with the conventional WLS method, and it is revealed that the accuracy of the proposed model is remarkably higher than the WLS method with the same $\mu$PMU numbers. The proposed algorithm can provide a reliable solution for the allocation of measuring devices in distribution networks so that the best possible locations are determined according to the network operator's preferences. In order to consider renewable power resources as distributed generation in an active distribution network, the proposed algorithm will be modified to model the uncertainty of these power generation resources in future studies. To this end, the solution procedure should be upgraded for efficient performance in the presence of uncertain scenarios. Moreover, combining other terms into objective function (multi-objective) can be the subject of complementary studies to improve the overall operation of distribution networks by enhanced monitoring.

## References

[1] M. S. F. Jabari and M. Zeraati, "A novel meter placement algorithm based on monte carlo coupled state estimation and iterative nonlinear mesh adaptive direct search," *J. Oper. Autom. Power Eng.*, vol. In Press, pp. 1–36, 2023.

[2] F. Jabari, M. Zeraati, M. Sheibani, and H. Arasteh, "Robust self-scheduling of pvs-wind-diesel power generation units in a standalone microgrid under uncertain electricity prices," *J. Oper. Autom. Power Eng.*, 2022.

[3] M. Pau, F. Ponci, A. Monti, C. Muscas, and P. A. Pegoraro, "Distributed state estimation for multi-feeder distribution grids," *IEEE Open J. Instrum. Meas.*, vol. 1, pp. 1–12, 2022.

[4] M. Zeraati, M. R. Sheibani, F. Jabari, and E. Heydarian-Forushani, "A novel state estimation method for distribution networks with low observability based on linear ac optimal power flow model," *Electr. Power Syst. Res.*, vol. 228, p. 110085, 2024.

[5] S. Tiwari, A. Kumar, and V. Basetti, "Multi-objective micro phasor measurement unit placement and performance analysis in distribution system using nsga-ii and promethee-ii," *Meas.*, vol. 198, p. 111443, 2022.

[6] A. A. M. Raposo, A. B. Rodrigues, and M. da Guia da Silva, "Robust meter placement for state estimation considering distribution network reconfiguration for annual energy loss reduction," *Electr. Power Syst. Res.*, vol. 182, p. 106233, 2020.

[7] B. P. Chintala and D. M. V. Kumar, "Multi-objective hybrid decomposition and local dominance based meter placement for distribution system state estimation," *IET Gener. Transm. Distrib.*, vol. 14, no. 20, pp. 4416–4425, 2020.

[8] R. Madbhavi, B. Natarajan, and B. Srinivasan, "Meter placement approaches for matrix completion-based distribution system state estimator," *Electr. Power Syst. Res.*, vol. 213, p. 108687, 2022.

[9] M. P. Anguswamy, M. Datta, L. Meegahapola, and A. Vahidnia, "Optimal micro-pmu placement in distribution networks considering usable zero-injection phase strings," *IEEE Trans. Smart Grid*, vol. 13, no. 5, pp. 3662–3675, 2022.

[10] M. Moradi-Sepahvand, E. Mashhour, and S. S. Mortazavi, "Optimal placement of a combination of single-phase and three-phase pmus for observability of smart distribution networks with asymmetrical structure," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 592–601, 2019.

[11] A. B. Pengwah, Y. Z. Gerdroodbari, R. Razzaghi, and L. L. H. Andrew, "Topology identification of distribution networks with partial smart meter coverage," *IEEE Trans. Power Delivery*, vol. 39, no. 2, pp. 992–1001, 2024.

[12] P. Buason, S. Misra, S. Talkington, and D. K. Molzahn, "A data-driven sensor placement approach for detecting voltage violations in distribution systems," *Electr. Power Syst. Res.*, vol. 232, 2024.

[13] C. B. Prasad and D. V. Kumar, "Meter placement in active distribution system using objective discretization and indicator-based multi-objective evolutionary algorithm with adaptive reference point method," *J. Inst. Eng. India Ser. B*, vol. 103, no. 3, pp. 887–901, 2022.

[14] H. Zhang and T. Zufferey, "Studying the impact of smart meter placement on low-voltage grid state estimation," in *2022 Int. Conf. Smart Energy Syst. Technol.*, pp. 1–6, IEEE, 2022.

[15] P. Paruta, T. Pidancier, M. Bozorg, and M. Carpita, "Greedy placement of measurement devices on distribution grids based on enhanced distflow state estimation," *Sustainable Energy Grids Networks*, vol. 26, p. 100433, 2021.

[16] H. G. Abood, V. Sreeram, and Y. Mishra, "An incremental meter placement method for state estimation considering collinear measurements and high leverage points," *Int. J. Smart Sens. Intell. Syst.*, vol. 13, no. 1, pp. 1–12, 2020.

[17] D. M. Ferreira, P. M. Carvalho, and L. A. Ferreira, "Optimal meter placement in low observability distribution networks with der," *Electr. Power Syst. Res.*, vol. 189, p. 106707, 2020.

[18] V. Zamani and M. E. Baran, "Meter placement for conservation voltage reduction in distribution systems," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 2109–2116, 2017.

[19] M. Zeraati, M. Shabanzadeh, M. R. Sheibani, and F. Jabari, "Meter placement algorithms to enhance distribution systems state estimation: Review, challenges and future research directions," *IET Renewable Power Gener.*, vol. 16, no. 15, pp. 3422–3444, 2022.

[20] B. P. Chintala and D. V. Kumar, "Multiobjective meter placement in active distribution system state estimation using inverse-model-based multilabel gaussian classification with adaptive reference point method," *Int. Trans. Electr. Energy Syst.*, vol. 31, no. 8, p. e12935, 2021.

[21] F. G. Duque, L. W. de Oliveira, E. J. de Oliveira, and J. C. de Souza, "A cost-benefit multiobjective approach for placement of meters in electrical distribution systems," *Electr. Power Syst. Res.*, vol. 191, p. 106897, 2021.

[22] E. T. Ghadikolaee, A. Kazemi, and H. A. Shayanfar, "Novel multi-objective phasor measurement unit placement for improved parallel state estimation in distribution network," *Appl. Energy*, vol. 279, p. 115814, 2020.

[23] A. H. Marzouni, A. Zakariazadeh, and P. Siano, "Measurement devices allocation in distribution system using state estimation: A multi-objective approach," *Int. Trans. Electr. Energy Syst.*, vol. 30, no. 8, p. e12469, 2020.

[24] T. C. Xygkis and G. N. Korres, "Optimized measurement allocation for power distribution systems using mixed integer sdp," *IEEE Trans. Instrum. Meas.*, vol. 66, no. 11, pp. 2967–2976, 2017.

[25] M. Pau, M. Ginocchi, D. Pacheco, F. Ponci, and A. Monti, "A meter placement strategy for distribution grids based on cost–benefit analysis," *IEEE Trans. Instrum. Meas.*, vol. 73, pp. 1–12, 2024.

[26] M. Mahdavi, H. H. Alhelou, and P. Cuffe, "Test distribution systems: network parameters and diagrams of electrical structural," *IEEE Open Access J. Power Energy*, vol. 8, pp. 409–420, 2021.