Journal of Operation and Automation in Power Engineering

Vol. 14, No. 4, Dec. 2026, Pages: 267-274

http://joape.uma.ac.ir

Research Paper

Coordinated Distributed Security Constrained Unit Commitment with Frequency Deviation Control for High Renewable Penetration Low Inertia Power Grids

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Abstract— *Among different renewable resources, wind power and solar photovoltaics (PVs) have the most desirable technical and economic prospects. However, the significant penetration of such low inertia power plants in power grids causes a decrease in the system's total inertia and thereby leads to a decrease in system frequency deviation (SFD) more than the acceptable range. This paper presents a distributed (D-SCUC) problem in which system frequency deviation is considered as a constraint to prevent system frequency deviation more than the pre-determined level. With the proposed method, the system partitions into several areas wherein a SCUC problem is separately solved, and the analytical target cascading (ATC) method is used to coordinate these sub-systems. To avoid the masking effect, a modified penalty function is used. A simple 6-bus network and the modified IEEE RTS 24-bus test system are used as the case study. The results show the effectiveness of the D-SCUC technique, especially in large power systems, and therefore, the system operator's concern about the system frequency is relieved.*

Keywords—Renewable energy sources, wind power, solar PVs, D-SCUC problem, low inertia, analytical target cascading, system frequency deviation.

NOMENCLATURE

$\Delta\omega$	Frequency deviation
ΔP_d	Power imbalance disturbance
$\gamma(m,n)$	Shared variables sending from area m to area n
$\rho_a^k(m,n)$	Coefficient of penalty function in iteration k
$\sigma_a^k(m,n)$	
F	Power fraction from HP turbine
h	Index of time (hour)
$I(u,h)$	Generator status (0: 'OFF, 1: 'ON')
K	Mechanical power gain factor
k	Governor reheat time constant
m,n	Index of areas
$M=2H$	Unit inertia
MSR	Maximum sustained ramp rate of unit g (MW/min)
$P(d,h)$	System demand at time h
$P(u,h)$	Power Generated of unit u at time h
$PD(h)$	Active power demand at time h
R	Governor droop
SZ_m	Set of areas next to the area m
u	Index of units
N_A	Number of areas
N_h	Number of periods under study
N_U	Number of units
$P_L(h)$	Limitation of Active power flow of Line in time h.
$R^{Dn}(u)$	Ramp-down power limit of unit u
$R^{up}(u)$	Ramp-up power limit of unit u
$T^{off}(g)$	Generator minimum down-time
$T^{on}(g)$	Generator minimum up-time
T_g	Governor reheat time constant
V_a	Voltage magnitude of bus a
$X^{off}(u,h)$	OFF time of unit g at time h
$X^{on}(u,h)$	ON time of unit g at time h
$X_{aa'}$	Reactance of line connecting buses a and a'

Received: 30 Aug. 2024
Revised: 03 Jan. 2025
Accepted: 10 Jan. 2025
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DOI: 10.22098/joape.2025.15739.2210

1. INTRODUCTION

One of the most important economic programs in power system is to reduce the operating costs of system ensuring the security of the power system. The SCUC problem is a discrete optimization problem with large dimensions and multiple and complex constraints. The existence of continuous and discrete variables and in some cases, nonlinear nature of objective function and constraints, increases the problem complexity. SCUC is not convex and is a non-deterministic polynomial hard (NP-hard) problem, i.e., the solution time may increase exponentially by increasing the dimensions of the problem (the number of variables) [1].

Development of distributed generation such as wind farms, small hydroelectric power plants, PV cells, and etc., for reducing environmental pollution, lack of asset resources for the construction of centralized power plants and a long period of return on investment, increasing the quality and security of power, energy price reduction, transmission lines losses decrement, creates new challenges to solve the SCUC problem. These resources increase the need to control the frequency of system for two main reasons: firstly, the unpredictable nature of most such resources and thereby the intermittent power injected to the network, secondly, with the low inertia characteristic of such resources as compared to the traditional power plants, leads to decrease the system total inertia which is in fact key factor in frequency regulation of power systems [2].

1.1. Review of articles

Lots of methods have been proposed to solve SCUC problem. Unit commitment (UC) problem was limited to solution of economic dispatch that is solved by using Crash-Kun-Tucker (KKT) conditions. Based on KKT conditions, several solutions for UC problem were established, such as lambda iteration method and gradient method [3, 4]. As systems became more complicated, most of presented methods are unable to solve the problem and meta-heuristic methods such as fuzzy logic, neural networks, genetic algorithm, and etc. are used to solve the SCUC problem [3–8]. In [9], a distributed method is proposed to solve SCUC. Another method for solving large optimization problems is to use alternating direction method of multipliers (ADMM) that uses partial updates for binary variables and for an easier solution in which the main problem is divided into several sub-problems [10]. The optimal power flow problem has been solved in [11] with ADMM method and in [12] with distributed method and using auxiliary problem principle (APP) for large-scale power systems. The optimal power flow problem has also been solved with a distributed time limited algorithm method based on ADMM [13].

International Renewable Energy Agency IRENA has reported a significant growth of renewable energy based power plants indicating that power systems is experiencing fundamental changes [14] and defined programs for the development of renewable energy power plants in different countries indicate the growth of their number and capacity in coming years. The inertia of a system consists of the inertia of rotating components connected to it. In power systems, the main part of inertia is usually created from the inertia of the rotors of synchronous generators. In addition to the rotor inertia of generators, rotating loads can also increase the network inertia. Rotating parts of the system initially try to compensate imbalance by absorbing or releasing part of their stored energy [15]. Frequency fluctuations with sudden changes in load and generation are more severe in networks with lower inertia. One of the methods to prevent sudden changes in frequency in low inertia systems is to consider frequency constraint in unit commitment of system.

In order to achieve the optimal global response, the planning of the primary frequency response resources must be simultaneously considered in the planning of the system's energy supply [16]. So, before using renewable resources, a suitable substructure should be created for optimal simultaneous planning of ancillary services and energy [17, 18]. In one of the first researches, the problem of optimal power flow of units is solved by taking into account the frequency limitations, in which, a non-linear model with provision of reservation is proposed to prevent the network frequency drop [17]. In [19], new constraints add to SCUC problem framework to represent the frequency response of system. An analytical model for the minimum frequency of system in thermal multi-machine power systems is proposed and an effective linearization method to linearize the nonlinear function that represents the minimum frequency of the system. This method can be used to combine distributed generation in power systems with considering the system frequency constraints.

1.2. Contribution

Since power systems are interconnected and different areas are connected by tie lines, and for problems such as SCUC, system must be considered completely, in the proposed method to reduce the computation time, an SCUC problem is separately solved for each part. After that the target and response variables are introduced and the sub-problems are related to the main problem.

The SCUC problem due to the presence of distributed generations with low inertia, in order to maintain the frequency in the acceptable range, using the principles of primary frequency control operation in the interconnected network with two or more areas that connected to each other by a transmission line.

Equations used to satisfy the frequency constraint are steady state deviation frequency error, acceptable minimum and maximum frequency and the rate of change of frequency during sudden change in load or loss of generation, which are dependent on dynamic parameters such as governor droop, inertia, load damping factor and mechanical power gain factor of units.

2. The effects of distributed generations with low inertia on system frequency

Solar and wind power plants are among the most important renewable power plants. The electric current generated by solar panels is DC and an inverter is needed for connecting them to an AC network. Since these power plants and the inverters connected to them don't have rotating parts, it leads a reduction in the inertia of network.

Wind power plants, due to the use of electronic power interfaces, reduce inertia and consequently reduce the security margin of system frequency. Studies have been carried out to solve this problem. For example, in DFIG based wind turbines, this problem is solved by improving the inertial response of these resources immediately after occurrence of fault. In a method by applying new controls in wind turbines, active power is injected to the grid by wind turbine right after the occurrence of a fault, but this method is challenging, so that after the occurrence of a fault and the injection of energy into the network, turbine speed of these power plants will experience a sudden drop, which can put mechanical tension that damages various parts wind turbine [20]. In the initial design of this type of wind power plants, more capacity should be considered to provide the required power after the fault occurrence, which in most cases leads to many operational limitations. A method is presented to limit the presence of DFIG and permanent magnet wind power plants in the network structure [21]. In second method, planning of participation of power plant units is done based on dynamic characteristics of the units so that the power plants in the network have ability to control the frequency. Fig. 1 shows the frequency changes in case of a sudden loss of a generator.

Inertia response is the response of the system immediately after the incident. During the initial period, the generator governor still does not respond due to the frequency dead band.

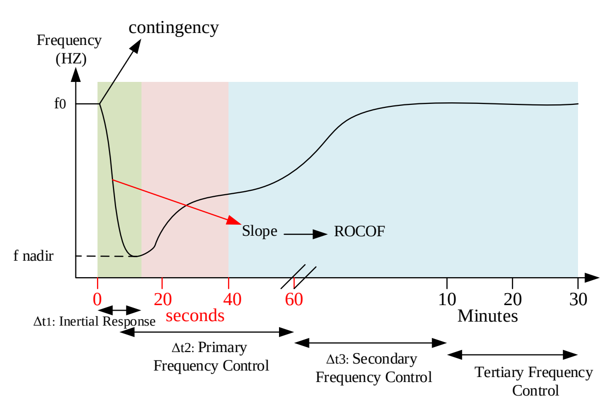

Fig. 1. Transient frequency of the power system after sudden loss of generator [22].

3. Formulation of SCUC problem in the presence of low inertia wind and solar PVs

3.1. Objective function of problem

Objective function of SCUC problem is a MINLP according to Eq. (1):

$$Min \sum_{h=1}^{N_H} \sum_{u=1}^{N_u} [C_u(p(u,h))I(u,h) + SU(u,h) + SD(u,h)] \quad (1)$$

The unit cost is usually a quadratic nonlinear function according to Eq. (2), which complicates the problem. The parameters are the same as [21].

$$\begin{aligned} &C_u(P(u,h)) = a_u v(u,h) + b_u P(u,h) + \\ &c_u P^2(u,h) \qquad \forall u \in N_u, \forall h \in N_h \end{aligned} \quad (2)$$

3.2. Constrains of problem

- **Active power balance of system**

$$\sum_{u=1}^{N_u} P(u,h)I(u,h) = P_D(h) \qquad h = 1, ..., N_h \quad (3)$$

- **Reserve service**

Reserve services are designed to respond to unpredictable and large events that can threaten the stability of the system. The spinning reserve is allocated according to specific rules, based on which the reserve number of different units is determined. The reserve may be equal to a percentage of the predicted maximum load, or it may have the ability to cover the shortage due to the loss of the largest unit at a certain time.

Spinning reserve of system

$$\begin{aligned} &\sum_{u=1}^{N_u} r_s(u,h)I(u,h) \geq R_s(h) \qquad h = 1, ..., N_h \\ &r_s(u,h) = \min\{10 \times MSR(u), P_{gmax}(u,h) - P(u,h)\} \end{aligned} \quad (4)$$

Operation reserve of system

$$\sum_{u=1}^{N_u} r_o(u,h)I(u,h) \geq R_o(h) \qquad h = 1, ..., N_h \quad (5)$$

- **Limitation of power generation of units**

$$\begin{aligned} &P_{u\min}(u)I(u,h) \leq P(u,h) \leq P_{g\max}(u)I(u,h) \\ &u = 1, ..., N_u \quad , \quad h = 1, ..., N_h \end{aligned} \quad (6)$$

- **Minimum ON and OFF time of units**

Minimum on-time when the unit is active, it cannot be stopped immediately and the minimum time required to be off. Minimum off-time the minimum time required when the unit is taken out of the circuit before it can be put back into the circuit.

$$[X^{on}(u,h) - T^{on}(u)] \times [I(u,h-1) - I(u,h)] \geq 0 \quad (7)$$

$$\left[X^{off}(u,h-1) - T^{off}(u,h)\right] \times [I(u,h-1) - I(u,h)] \geq 0 \quad (8)$$

- **Ramp constraints**

A unit often cannot change its output power immediately. Ramp constraints are used to model this constraint.

$$[P(u,h) - P(u,h-1)] \leq R^{up}(u) \quad (9)$$

$$[P(u,h-1) - P(u,h)] \leq R^{Dn}(u) \quad (10)$$

- **Line flow equations**

DC power flow equation define line flow between buses a, a'.

$$P_L = \frac{1}{X_{aa'}} \times (\theta_a - \theta_{a'}) \quad (11)$$

- **The maximum permissible transmission power of the lines**

Because of technical reasons, each line has a power passing range so that if the power passing through the lines exceeds this range, the conductors of the line will suffer compression (filling).

$$P_L^{\min} \leq P_L(h) \leq P_L^{\max} \qquad t = 1, ..., N_h \quad (12)$$

- **Frequency limit constraints**

Network frequency control, especially right after the occurrence of various contingencies or faults, is an important issue in recent power systems with high penetration level of low inertia RESs. The frequency-related limitations include the limitation in RoCoF and the steady-state error of the frequency, which are obtained by using low-order frequency response model of system, as presented in Fig. 2.

This model, which includes dynamic characteristics of power plant units such as droop characteristics, inertia, generation coefficient and mechanical gain, can be used to estimate the frequency behavior of a power system to sudden load or generation disturbances [21].

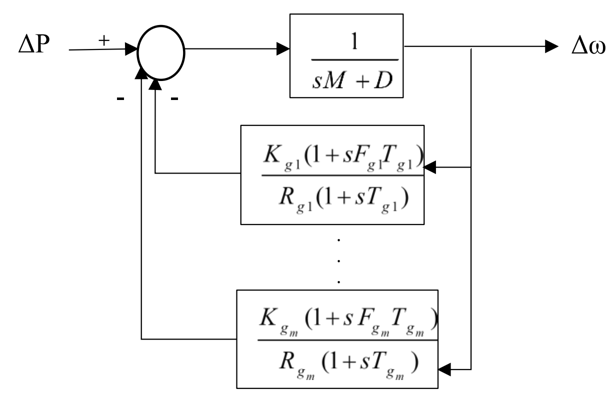

Fig. 2. Low order frequency response model of system.

Transfer function of system in Fig. 2 is defined as the following relationship [21].

$$\frac{\Delta\omega}{\Delta P_d} = \frac{1}{(Ms + D) + \sum_{g \in N_g} \left[\frac{K_g}{R_g} \frac{1+sF_gT_g}{1+sT_g}\right]} \quad (13)$$

Assuming D=0 and $T_{g_1} = ... = T_{g_m} = 0$:

$$\frac{\Delta\omega}{\Delta P_d} = \frac{1}{Ms} \frac{1 + sT}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (14)$$

where in:

$$\omega_n = \sqrt{\frac{R_T}{MT}} \quad (15)$$

$$\zeta = \frac{M + F_T T}{2\sqrt{R_T MT}} \quad (16)$$

The parameters of Eqs. (15) and (16) are defined as follows:

$$M = \frac{\sum\limits_{g \in N_g} M(g)P(g)}{\sum\limits_{g \in N_g} P(g)} \tag{17}$$

$$F_T = \sum_{g \in N_g} \frac{K_g F_g}{R_g} \tag{18}$$

$$R_T = \sum_{g \in N_g} \frac{K_g}{R_g} \tag{19}$$

Assuming the step function as a disturbance in the input $\Delta P_d = -\Delta p/s$, time domain response $\Delta\omega$ is as follows:

$$\begin{aligned}&\Delta\omega(t) = \\ &\tfrac{\Delta P}{MT\omega_n^2} - \tfrac{\Delta P}{M\omega_r} e^{-\zeta\omega_n t}(\sin\omega_r t - \tfrac{1}{\omega_r T}\sin(\omega_r t + \varphi))\end{aligned} \tag{20}$$

where in:

$$\omega_r = \omega_n\sqrt{1-\zeta^2} \tag{21}$$

$$\varphi = \sin^{-1}\left(\sqrt{1-\zeta^2}\right) \tag{22}$$

By assuming the occurrence of the most frequency drop in t^z, the following equation can be used to calculate t^z, and by substituting t^z in Eq. (20), the minimum frequency (maximum frequency drop) is obtained from Eq. (24).

$$\frac{d\Delta\omega(t)}{dt} = 0 \to t^z = \frac{1}{\omega_r}\tan^{-1}\left(\frac{\omega_r}{\zeta\omega_n - 1/T}\right) \tag{23}$$

$$\Delta\omega\left(t^z\right) = -\frac{\Delta P}{R_T}\left(1 + e^{-\zeta\omega_n t^z}\sqrt{\frac{T(R_T - F_T)}{M}}\right) \tag{24}$$

ROCOF is also obtained from Eq. (25).

$$\left.\frac{d\omega}{dt}\right|_{t=0^+} = -\frac{\Delta P}{M} \tag{25}$$

The following equation used to calculate the frequency steady state error.

$$\omega_{ss} = -\frac{\Delta P}{R_T} \tag{26}$$

4. Solving SCUC problem with parallel computing method

By increasing the dimensions of the SCUC problem, number of combinations different states of the units being off and on, increases exponentially and the problem solving takes time. various methods are used to improve the SCUC problem solving [23].

Objective function in Eq. (1) and constraint Eqs. (3)-(12) can be rewritten as Eq. (27):

$$\begin{aligned}&\min \sum_{u=1}^{Nu_m}\sum_{h=1}^{N_h} F_{hm}(P(g,h)I(g,h)) + \\ &SU(g,h) + SD(g,h) \qquad \forall NG_m,\ P \in m \\ &s.t. \quad g_m(P_m, \theta_m, I_m) \le 0 \\ &\qquad h_m(P_m, \theta_m, I_m) = 0\end{aligned} \tag{27}$$

The variables P_m, θ_m, I_m and respectively, are a set of output power of the units, angles of the buses and on or off status of the generation units related to the area m in different time interval. constraints associated with area m (g_m and h_m) are among the conventional generation and network constraints [9]. Assuming that area m and area n are connected, Eq. (28) should be considered in L-SCUC problem. By using separate penalty function, constraint Eq. (28) is relaxed in local objective function.

$$\kappa(m,n,h) = \gamma(m,n,h) - \gamma(n,m,h) = 0 \qquad \forall h \tag{28}$$

So, the L-SCUC problem of area m is expressed as follows:

$$\begin{aligned}&Min \sum_{u=1}^{Nu_m}\sum_{h=1}^{N_h} F_{gm}\left(\tilde{P}(u,h)\right)\tilde{I}(u,h) + SU(u,h) + \\ &SD(u,h) + \sum_{h=1}^{N_h}\sum_{n \in SZ_m} \varphi(\kappa(m,n,h)) \\ &s.t. \quad g_m(\tilde{P}_m, \tilde{\theta}_m, \tilde{I}_m, S_m) \le 0 \\ &\qquad h_m(P_m, \theta_m, I_m, S_m) = 0 \\ &\forall S(m,n,h) \in \{\gamma(m,n,h), \gamma(n,m,h)\} \quad \forall n \in SZ_m, \forall h\end{aligned} \tag{29}$$

Due to the possibility of connection of area m with several areas, index n in Eq. (29) can show all areas that have shared variables with area m. An augmented Lagrange is used to model the penalty function φ in Eq. (29). So, Eq. (29) is rewritten as Eq. (30):

$$\begin{aligned}&Min \sum_{u=1}^{Nu_m}\sum_{h=1}^{N_h} F_{um}\left(\tilde{P}(u,h)\right)\tilde{I}(u,h) + \\ &SU(u,h) + SD(u,h) \\ &+ \sum_{h=1}^{N_h}\sum_{n \in SZ_m} (\rho(m,n,h)\left((\gamma(m,n,h) - \gamma(n,m,h)\right) + \\ &\|\sigma(m,n,h) \circ (\gamma(m,n,h) - \gamma(n,m,h))\|_2^2)\end{aligned} \tag{30}$$

where the symbol o indicates the Hadamard multiplication or the linear multiplication of two vectors. The penalty coefficients are updated during the iterative solution. The objective function of the L-SCUC problem of area m, Eq. (30) is rewritten as Eq. (31). Target variables $\theta_a(m,n,h)$ and $\theta_{a'}(m,n,h)$ in the penalty function are determined, but the response variables $\theta_a^*(n,m,h)$ and $\theta_{a'}^*(n,m,h)$ are received from other areas.

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$$\begin{aligned}&Min \sum_{u=1}^{Nu_m}\sum_{h=1}^{N_h} F_{um}\left(\tilde{P}(u,h)\right)\tilde{I}(u,h) + \\ &SU(u,h) + SD(u,h) + \\ &\sum_{h=1}^{N_h}\sum_{(a,a') \in SZ_m}\left\{\begin{array}{l}\rho_a(m,n,h)\left((\theta_a^*(n,m,h) - \theta_a(m,n,h)\right) + \\ \|\sigma_a(m,n,h)^\circ(\theta_a^*(n,m,h) - \theta_a(m,n,h))\|_2^2 + \\ \rho_{a'}(m,n,h)\left((\theta_{a'}^*(n,m,h) - \theta_{a'}(m,n,h)\right) + \\ \|\sigma_{a'}(m,n,h)^\circ(\theta_{a'}^*(n,m,h) - \theta_{a'}(m,n,h))\|_2^2\end{array}\right\}\end{aligned} \tag{31}$$

As seen in Fig. 3, the sub-problems are only connected to the main problem and there is no connection between the sub-problems.

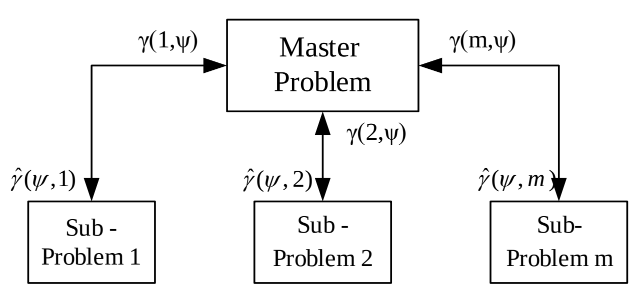

Fig. 3. Parallelizing the distributed multi-area SCUC.

Table 1. SCUC results Without Frequency (WoF) constraints and With Frequency (WF) constraint for the 6-bus test system.

Hour		1	2	3	4	5	6	7	8	9	10	11	12
G1	WoF	140	140	133.7	129.7	106.1	109.5	131.4	122.5	120.8	131.4	140	140
	WF	140	140	133.7	100					100	140	140	140
G2	WoF												
	WF					11.1	14.5	46.4	37.5				
G3	WoF	25.2	13.2	10				10	10	10	25.6	45.6	56.1
	WF	25.2	13.2	10	29.7	95	95	95	95	30.8	17	45.6	56.1
Hour		13	14	15	16	17	18	19	20	21	22	23	24
G1	WoF	140	140	140	140	140	140	140	140	140	140	128.3	140
	WF	140	140	140	140	140	140	140	140	140	140	140	140
G2	WoF						10	10					
	WF						10	10					
G3	WoF	50.2	53.6	63.9	74.8	84	86.7	91	85.4	73.3	71.7	51.7	41.6
	WF	50.2	53.6	63.9	74.8	84	86.7	91	85.4	73.3	71.7	40	41.6
Total cost	WoF	**$62330**											
	WF	**$66150**											

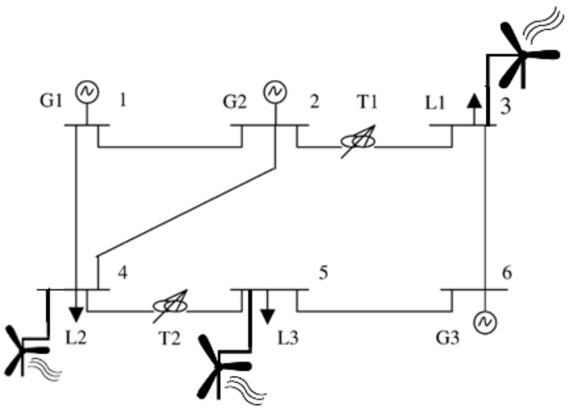

Fig. 4. 6-bus test system [24].

Table 2. Comparison of centralized and distributed method for 6-bus and 24-bus system with and without frequency constraints.

		Centralized SCUC		D-SCUC	
		Calculation time (s)	Cost ($)	Calculation time (s)	Cost ($)
IEEE 6-bus system	WoF	0.21	62330	0.25	62335.54
	WF	0.32	66150	0.37	66158.87
IEEE 24-bus system	WoF	0.51	209226	0.45	209250
	WF	0.75	212582	0.6	212600

Algorithm:

Step 1) The iteration index is set to k=1, initial values of virtual shared variables $\hat{\theta}_a^*(\psi,m,h)$, $\hat{\theta}_{a'}^*(\psi,m,h)$ and penalty coefficients $\rho_a^k(m,\psi,h)$, $\rho_{a'}^k(m,\psi,h)$, $\sigma_a^k(m,\psi,h)$ and $\sigma_{a'}^k(m,\psi,h)$ are selected.

Step 2) In parallel, the m subproblem of SCUC (m=1:NZ) is solved with the variables $\theta_a^k(m,\psi,h)$ and $\theta_{a'}^k(m,\psi,h)$. The variables $\theta_a^{*k-1}(m,\psi,h)$ and $\theta_{a'}^{*k-1}(m,\psi,h)$ are obtained from main problem in k-1 iteration.

Step 3) The main problem is solved with the variables $\hat{\theta}_a^{k-1}(\psi,m,h)$ and $\hat{\theta}_{a'}^{k-1}(\psi,m,h)$ with the values $\theta_a^{*k}(m,\psi,h)$ and $\theta_{a'}^{*k}(m,\psi,h)$ that obtained in step 2.

Step 4) The stopping criterion is checked. If it is not met, go to step 5; Otherwise, it converges and the desired result is obtained and the solution is stopped.

Conditions Eqs. (32) and (33) are necessary and sufficient conditions.

$$\left|\theta_a^k(m,\psi,h)-\hat{\theta}_a^k(\psi,m,h)\right|\leq\varepsilon_1 \quad \forall m\ ,\forall h \tag{32}$$

$$\left|\theta_{a'}^k(m,\psi,h)-\hat{\theta}_{a'}^k(\psi,m,h)\right|\leq\varepsilon_1 \quad \forall m\ ,\forall h \tag{33}$$

Step 5) set k=k+1 and the values of coefficients $\rho_a^k(m,\psi,h)$, $\rho_{a'}^k(m,\psi,h)$, $\sigma_a^k(m,\psi,h)$ and $\sigma_{a'}^k(m,\psi,h)$ are obtained from Eqs. (34) to (35) and go to step (2).

$$\begin{aligned}&\rho_a^k(m,\psi,h)=\rho_a^{k-1}(m,\psi,h)+\\&2(\sigma_a^{k-1}(m,\psi,h))^2(\theta_a^{k-1}(m,\psi,h)-\hat{\theta}_a^{k-1}(m,\psi,h))\end{aligned} \tag{34}$$

$$\begin{aligned}&\rho_{a'}^k(m,\psi,h)=\rho_{a'}^{k-1}(m,\psi,h)+\\&2(\sigma_{a'}^{k-1}(m,\psi,h))^2(\theta_{a'}^{k-1}(m,\psi,h)-\hat{\theta}_{a'}^{k-1}(m,\psi,h))\end{aligned} \tag{35}$$

$$\sigma_a^k(m,\psi,h)=\lambda\sigma_a^{k-1}(m,\psi,h) \tag{36}$$

$$\sigma_{a'}^k(m,\psi,h)=\lambda\sigma_{a'}^{k-1}(m,\psi,h) \tag{37}$$

In order to converge, λ should be bigger than one.

5. Simulation Results

The propped frequency constraint D-SCUC is studied based on a simple 6-Bus network as well as the modified IEEE RTS 24-Bus test system. For each case study, SCUC problem is solved in the form of centralized SCUC and also distributed SCUC (D-SCUC) with and without frequency constraints.

5.1. 6-Bus test system

The 6-bus test system is shown in Fig. 4. The system data is given in [24] and dynamic data of generators, wind generation data, and contribution of each load in buses is given in [19].

Assuming a 10% sudden load increase, using Eqs. (13)-(26), the frequency changes of each generator for different conditions are shown in Figs. 5-a to 5-d.

First, SCUC problem is solved without frequency constraints (Eqs. (1)-(12)), and results are presented in Table 1. There are hours, for example hour 4 and 5, where only one unit is ON. Assuming a 10% load increase (or 10% reduction in generation) for different combinations of online units, for example $\{G_1\}$, $\{G_1, G_2\}$ and $\{G_1, G_2, G_3\}$, the minimum speed of each generator is checked, the results in Fig. 5-a, according to Eq. (13) show that when three generators are ON, frequency drop is less than being one or two are ON and Fig. 5-c shows that less inertia led to more frequency drop.

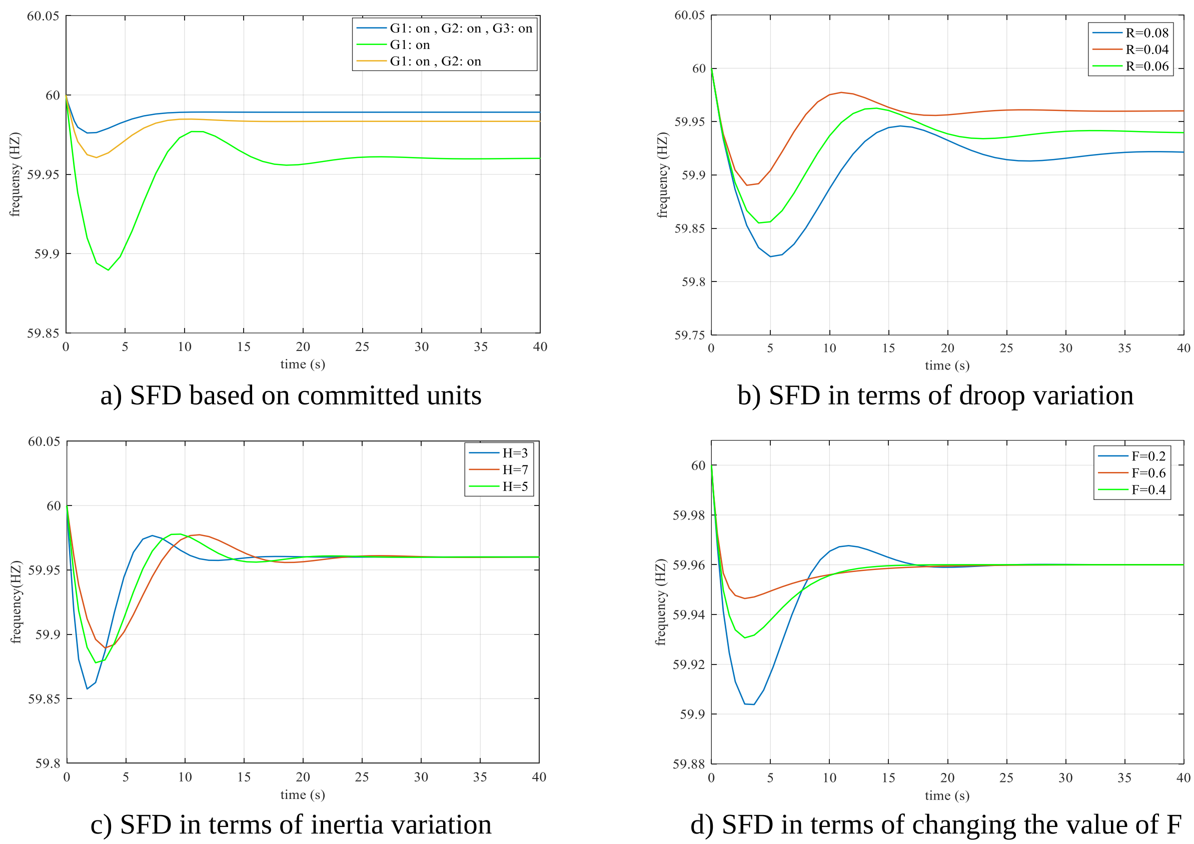

Fig. 5. SFD to a sudden 10% load increase.

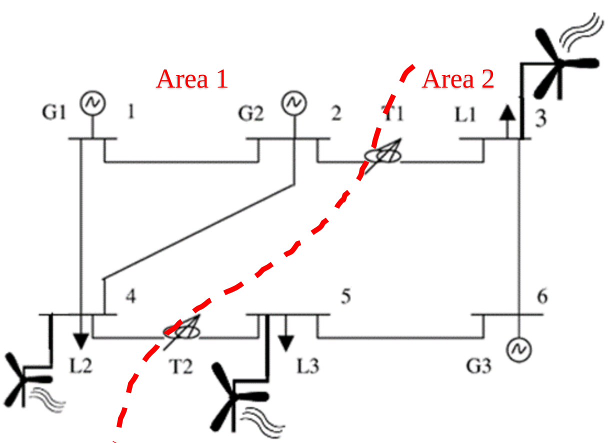

Fig. 6. 6-bus test system divided in two areas.

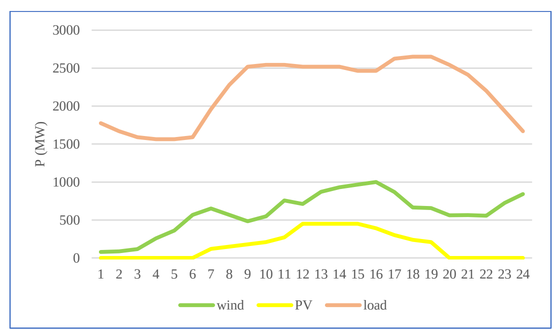

Fig. 7. Load profile and generated power of DGs connected to 24-bus test system.

The constraints Eqs. (25)-(26) are added to SCUC formula and maximum acceptable ROCOF is supposed to be 0.1 Hz/s and maximum steady state frequency drop is also supposed to be 0.5 Hz. Table 1 show the results. Total operating cost for the first mode is $62,330 and in second mode is $66,150.

Table 1 are the result of solving SCUC in a centralized method. In following, according to Fig. 6, the system is partition into two areas and the problem is solved in a distributed way (D-SCUC).

The line between buses 2 and 3, as well as tie line between buses 4 and 5 and connects these two areas. The initial values $\hat{\theta}_a^{*0}(\psi, m, h)$ and $\hat{\theta}_{a'}^{*0}(\psi, m, h)$ are set to zero. (Due to the existence of 2 areas, m includes the values of 1 and 2 and θ_a, the voltage angles of buses 2 and 4 and $\theta_{a'}$, the voltage angles of buses 3 and 5 and are sent from the main problem to sub-problems 1 and 2). Coefficient of penalty function, $\rho_a^0(m, \psi, h) = \rho_{a'}^0(m, \psi, h) = \sigma_a^0(m, \psi, h) = \sigma_{a'}^0(m, \psi, h) = 100$ and $\lambda = 1$, also the convergence coefficient ε is 0.004.

SCUC sub-problems of areas 1 and 2 can be solved simultaneously in parallel and each of them sends four shared variable values, $\theta_2(1, \psi, h)$, $\theta_3(1, \psi, h)$, $\theta_4(1, \psi, h)$, $\theta_5(1, \psi, h)$, $\theta_2(2, \psi, h)$, $\theta_3(2, \psi, h)$, $\theta_4(2, \psi, h)$, $\theta_5(2, \psi, h)$ to the main problem.

After the first iteration, the stopping criteria Eqs. (34)-(36) are not satisfied, so this iteration process continues so long as the convergent result is found after 18 iterations, and the results are the same as those taken in Table 1. The total cost of system operation in D-SCUC (sum of sub-problems and main objectives) is $62,335.54, which is almost equal to the SCUC centralized method result ($62,330).

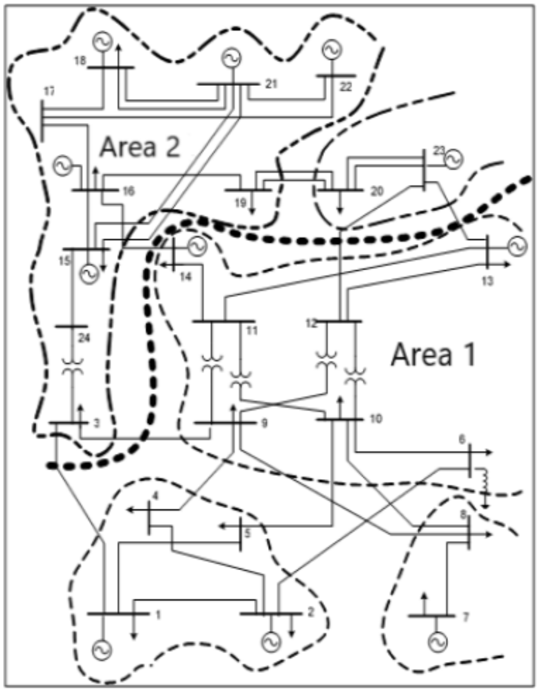

Fig. 8. 24-bus test system divided into two areas.

5.2. IEEE RTS 24-bus test system

The modified IEEE RTS 24-Bus test system consists of 12 power plants (33 units), 34 transmission lines and a peak load of 2650 MW. The system data is given in [25] and the dynamic data are taken from [26]. In order to find the effects of reducing inertia resulting from the use of wind power and solar PVs, it is assumed that 3 solar PVs are connected to buses 5, 14 and 20 and 5 wind power plants are connected to buses 6, 7, 16, 18 and 21.

The profile of the total generation capacities of solar, wind and load power are shown in Fig. 7. With this assumption, for example, during peak hours, about 50% of the required power is supplied by wind power and PVs.

Similar to the previous section, the first case is without frequency (WoF) constraint and the second case is with frequency (WF) constraint. To consider the frequency constraint, Eqs. (25) and (26) are used, and upper limit of ROCOF is 0.1 Hz/second and upper limit of steady state frequency drop is 0.5 Hz. With Frequency (WF) constraint, the total operation cost (SCUC cost) increases from $209,226to212,582$, an increase of 1.6%. Despite a slight increase in operating costs, but the admissible frequency is satisfied during the operating period.

In order to solve D-SCUC, according to Fig. 8, the system is partition into two areas and the problem is solved in a distributed way and the results are presented.

These two areas are connected through five lines. i.e., line 1-3, 3-9, 14-16, 12-23 and 13-23. So, the voltage angles of buses 1, 3, 9, 14, 16, 12, 13, and 23 are eight variables between subproblems of areas 1 and 2. these subproblems solve in parallel and each area sends eight variables to the main problem. This algorithm reaches the optimal solution after 26 iterations.

The obtained solution shows that the results of D-SCUC are almost equal to those obtain in centralized method (SCUC). Table 2 shows the values of calculation time and operating cost of 6-bus and 24-bus test system in a SCUC and D-SCUC manner.

When the test system is small, the centralized algorithm performs better than the distributed one, but by increasing the size and complexity of the system, the D-SCUC algorithm performs better with lower computation time.

6. CONCLUSION

In this paper, the SCUC problem with low-inertia renewable resources is studied in the form of D-SCUC. The results show that by committing the thermal units with sufficient desired inertia every hour, the frequency limits can be met, and despite the increase of renewable resources, the transient stability of power systems can also be guaranteed. Also, a distributed SCUC algorithm reduces total calculation time by dividing the system into 2 or more areas, solving an L-SCUC for each area and then solving the main problem, especially for large systems. The results show that as the power system size is increased, the computation of the proposed D-SCUC becomes lower than that of the centralized one. This decrease in computation burden is remarkable in large-sized power systems, including thousands of buses and hundreds of generators. The following matters can be considered for future works:

1) Other functions can be used as penalty functions (Eq. (30)), for example, the exponential penalty function.
2) Considering the coherency of generators in dividing the system into areas. This matter is very helpful for the dynamic stability of power systems and especially the small signal stability.
3) Using the AC power flow equation (active and reactive power balance) in the D-SCUC, which in turn increases the complexity of the problem solution because D-SCUC will be a mixed integer nonlinear programming (MINLP) optimization problem. In this case, linear approximation, convex relaxation, etc., can be used to reach a mixed integer linear programming (MIP) optimization problem, which can be solved by available solvers such as CPLEX.
4) Using deep learning to solve D-SCUC to model and include frequency constraints in the SCUC problem to consider all of the system characteristics, ensuring locational frequency security.

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