Journal of Operation and Automation in Power Engineering
Vol. 3, No. 1, Winter & Spring 2015, Pages: 83-93
http://journals.uma.ac.ir/joape

# Probabilistic Multi Objective Optimal Reactive Power Dispatch Considering Load Uncertainties Using Monte Carlo Simulations

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**ABSTRACT**
*Optimal Reactive Power Dispatch (ORPD) is a multi-variable problem with nonlinear constraints and continuous/discrete decision variables. Due to the stochastic behavior of loads, the ORPD requires a probabilistic mathematical model. In this paper, Monte Carlo Simulation (MCS) is used for modeling of load uncertainties in the ORPD problem. The problem is formulated as a nonlinear constrained multi objective (MO) optimization problem considering two objectives, i.e., minimization of active power losses and voltage deviations from the corresponding desired values, subject to full AC load flow constraints and operational limits. The control variables utilized in the proposed MO-ORPD problem are generator bus voltages, transformers' tap ratios and shunt reactive power compensation at the weak buses. The proposed probabilistic MO-ORPD problem is implemented on the IEEE 30-bus and IEEE 118-bus tests systems. The obtained numerical results substantiate the effectiveness and applicability of the proposed probabilistic MO-ORPD problem.*

**KEYWORDS:** Monte Carlo simulation, Multi objective optimal reactive power dispatch, Real power loss, Voltage deviation.

## NOMENCLATURE

| | | | |
|---|---|---|---|
| k | k-th network branch that connects bus i to bus j | $J_{pu}$ | Normalized objective function |
| $i\ /\ j$ | Bus number where $i, j = 1, 2, \ldots, N_B$ | $J_r^{\max} / J_r^{\min}$ | Maximum /minimum value for r-th objective function |
| $g_k$ | Conductance of the line i-j | $w_1$ | Weight of objective 1 (real power loss) |
| $V_i$ | Voltage magnitude of bus i | $w_2$ | Weight of objective 2 (voltage deviation) |
| $\theta_i$ | Voltage angle at bus i | $PL$ | Real power loss |
| $x$ | Vector of dependent variables | $VD$ | Voltage deviation |
| $u$ | Vector of control variables | $N_D$ | Set of load bus |
| $J$ | Total objective function | $N_B$ | Number of buses |
| $J_1$ | First objective function (PL=Real power loss) | $\psi_k$ | Set of buses adjacent branch *k* |
| $J_2$ | Second objective function (VD=Voltage deviation) | $P_G$ | Active power in bus *i* |

Received: 27 Sep. 2014 Revised: 09 Jan. 2015
Accepted: 30 Jan. 2015
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| | |
|---|---|
| $Q_{Gi}$ | Reactive power generation in bus *i* |
| $P_{D_i}$ | Real power of the *i*-th bus |
| $Q_{D_i}$ | Reactive power of the *i*-th bus |
| $Y_{ij} = G_{ij} + jB_{ij}$ | ij-th element of system $Y_{bus}$ matrix |
| $S_\ell$ | power flow of $\ell$ -th transmission lin |
| $S_\ell^{max}$ | Maximum value of power flow of $\ell$ -th transmission line |
| $V_i^{min} / V_i^{max}$ | Minimum/ Maximum value for voltage magnitude of the i-th bus |
| $Q_{Gi}^{min} / Q_{Gi}^{max}$ | Minimum/ Maximum value for reactive power of the i-th bus |

## 1. INTRODUCTION

Optimal Power Flow (OPF) affects both security and economy of power systems, and hence, it has to be considered as an integral part of power system operation and planning studies. The OPF can be divided into two sub-problems, Optimal Reactive Power Dispatch (ORPD) and optimal real power dispatch [1], [2].

### 1.1. Literature review

The ORPD problem is a complex problem in power systems and has attracted great attention in recent years, because it is strongly related to both economy and security of the system [3]. In most cases, the aim of ORPD is to optimize the following objective functions:

- Minimization of the network real power losses (as an economical objective).
- Optimization of voltage profile of the network, by minimizing voltage deviations from their nominal values in the load buses.

The aforementioned objectives are attained by regulating generator bus voltages, VAr compensators switching on/off, and optimization of transformer tap settings, with respect to various operational constraints such as load flow equations [4].

The ORPD problem is extensively studied in the literature. For instance, management and rescheduling of reactive power support via an ORPD model is presented in [3]. The objective function in [3] is to maximize voltage stability margin, at the same time as taking care of the economic dispatch of active power, by rescheduling the reactive power injection of synchronous generators and synchronous condensers. An objective function which depends on a voltage stability index is offered in [4], for solving ORPD problem. A model for ORPD is proposed in [5] for minimization of total costs, including energy loss of transmission network and costs of adjusting the control devices. A solution for the ORPD problem by Particle Swarm Optimization (PSO) based on multi-agent systems is proposed in [6]. A Seeker Optimization Algorithm (SOA) is suggested for ORPD taking into consideration static voltage stability [7]. In [8], a harmony search algorithm is proposed for partially solution of ORPD problem. A steady-state voltage stability constrained ORPD model is studied in [9]. In [10], an evolutionary-based approximation is presented for ORPD solution. This approach uses a differential evolution algorithm in order to determination of optimal settings of ORPD control variables. A particle swarm optimization, combined with a feasible solution search used for dealing with the ORPD problem in the presence of Wind Farms (WF) is presented in [11]. The proposed approach optimizes the reactive power dispatch, considering the reactive power requirement at the WF point of connection. A hybrid approach based on the evolutionary planning and particle swarm optimiz-ation is proposed in [12] to solve the ORPD problem. In [13], the behavior of different constraint controlling methods such as superiority of feasible solutions, self-adaptive penalty, ε-constraint, stochastic ranking, and the ensemble of constraint handling techniques on ORPD are investigated. A heuristic algorithm is introduced in [14] by combining modified teaching learning algorithm and double differential evolution algorithm until to handle the ORPD problem. Furthermore, in [15], a reliable and effective algorithm based on hybrid modified imperialist competitive algorithm and invasive weed optimization is proposed for solving the ORPD problem. Furthermore, a hybrid algorithm combining firefly algorithm and Nelder mead simplex method is represented in [16] for solution of ORPD problem.

A number of literatures study Multi Objective ORPD problem, considering the uncertainties. For example, a strength Pareto evolutionary algorithm is proposed in [17] to handle the MO-ORPD. A hierarchical clustering algorithm was suggested to provide a representative and manageable Pareto-optimal set. In [18], a reformed version of NSGA-II was applied by incorporating controlled elitism and dynamic crowding distance strategies in NSGA-II. The approach is utilized to solve the MO-ORPD problem by minimizing real power loss and maximizing the system voltage stability. A hybrid fuzzy multi objective evolutionary algorithm for solving complicated MO-ORPD problem is reported in [19], which considers voltage stability. A well-organized genetic algorithm method for solution of MO-ORPD problem is represented in [20], which considers fuzzy goal programming in uncertain environment. In [21], an advanced teaching learning based optimization algorithm is presented to solve MO-ORPD problem by minimizing real power loss, voltage deviation and voltage stability index. Chaotic improved PSO based multi-objective optimization and improved PSO-based multiobjective optimization approaches are prop-osed in [22], for solving MO-ORPD problem. The objective functions considered are power losses and L index. In [23], a multi objective chaotic parallel vector evaluated interactive honey bee mating optimization is presented to find the optimal solution of MO-ORPD problem considering operational restrictions of the generators.

It should be noted that few references have considered the possible uncertainties in the MO-ORPD problem. For example, in [24], a chance-constrained programming formulation is proposed to solve the MO-ORPD problem that considers uncertain nodal power injections and random branch outages.

### 1.2. Contributions

It is observed from the above literature survey that the MO-ORPD problem has been solved so far with lots of intelligent algorithms, but the uncertainty of load demand which is key factor in MO-ORPD problem was not investigated so far.

Load forecasting is usually performed based on the past and future information of the system such as weather condition, temperature and demand requirement. But, because of the random nature of load, the nonlinear relationship between the load and climate, and lack of precision in the prediction of climate, always the forecasted real and reactive demands are inaccurate and a certain degree of prediction errors exist. Therefore, it is necessary to consider the uncertainty of loads in the MO-ORPD problem.

Since this paper focuses on the uncertainties associated with the load, it is assumed that the statistical model of loads are estimated or measured. Due to the composite load modeling in the ORPD problem, the load is modeled by normal Probability Distribution Function (PDF) with a known mean and standard deviation, which are obtained from historical data and load forecasting programs.

The following well suited objective functions are considered in this paper for MO-ORPD:

- Minimization of real power losses
- Minimization of voltage deviation from the corresponding nominal value.

The main contributions of this study are summarized as follows:

1- The effect of uncertain nature of loads is studied in the MO-ORPD problem. The normal PDF is used for this aim.
2- Monte Carlo simulation (MCS) is used to solve the probabilistic MO-ORPD problem.

The numerical results substantiate the superiority of the proposed probabilistic MO-ORPD model in comparison with the existing heuristic algorithms.

### 1.3. Paper organization

The rest of this paper is organized as follows: Sections 2 and 3 describe the ORPD and MO-ORPD problem formulations, respectively. Implementation of deterministic MO-ORPD, MCS-based MO-ORPD problems and numerical results are presented in Sec. 4. Finally, the findings and conclusions of this paper are summarized in Sec. 5.

## 2. RPD PROBLEM FORMULATION

A system operator usually has various objectives such as minimization of sum of system transmission loss, and voltage deviation of load buses from their desired values etc. These objective functions may conflict with each other. Hence, at the first, the confliction between them is investigated.

### 2.1. ORPD objective functions

In this paper, the objective functions are minimization of real power losses and voltage deviations from the corresponding nominal values, in load buses.

#### 2.1.1. Minimization of total real power losses

With the increasing rate of energy consumption, the amount of power losses are increased too, making the reduction of power losses as an important aim for system operators [25]. The active power losses can be expressed as follows [26].

$$J_1 = PL(x,u) = \sum_{\substack{k=1 \\ i,j\in\Psi_k}}^{N_L} g_k\left[V_i^2 + V_j^2 - 2V_iV_j\cos(\theta_i - \theta_j)\right] \tag{1}$$

#### 2.1.2 Minimization of voltage deviations at load bus

The second aim of ORPD problem is to maintain a proper voltage level at load buses. Any electrical equipment is designed for optimum operation at a nominal voltage. Any deviation from this specified voltage decreases its efficiency, damages it, and reduces its useful lifetime. Thus, the voltage profile of the system should be optimized. This is accomplished by minimization of sum of voltage deviations from the corresponding rated values at load buses. This objective function is stated as follows [27]:

$$J_2 = VD(x,u) = \sum_{i=1}^{N_D}\left|V_i - V_i^{spc}\right| \tag{2}$$

### 2.2. Constraints

#### 2.2.1. Equality constraints

The AC active/reactive power flows equations are expressed as follows.

$$P_{G_i} - P_{D_i} = V_i\sum_{j=1}^{NB}V_j\left[G_{ij}\cos(\theta_i-\theta_j) + B_{ij}\sin(\theta_i-\theta_j)\right]$$
$$Q_{G_i} - Q_{D_i} = V_i\sum_{j=1}^{NB}V_j\left[G_{ij}\sin(\theta_i-\theta_j) - B_{ij}\cos(\theta_i-\theta_j)\right] \tag{3}$$

#### 2.2.2. Operational limits

The generators reactive power output and bus voltages should be hold in a pre-specified interval, as follows:

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \tag{4}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{5}$$

Also, the line flow limits are as follows.

$$\left|S_\ell\right| \le S_\ell^{\max} \qquad \forall \ell \in NL \tag{6}$$

Besides, transformers' tap settings must be restricted by their lower and upper limits as follows:

$$T_i^{\min} \le T_i \le T_i^{\max} \tag{7}$$

## 3. MO-ORPID

Various methods are available to solve multi-objective optimization problems such as weighted sum approach [28], $\varepsilon$-constraint method [29] and evolutionary algorithms [30]. In this paper, the proposed multi-objective model of the MO-ORPD is solved using the weighted sum method. In this method, different weights are used for the conflicting objective functions to generate different Pareto optimal solutions. Hence, the overall objective function (which should be minimized) is the weighted sum of individual objective functions as follows:

$$\min\left[J(x,u)\right] = w_1 J_{1,pu}(x,u) + w_2 J_{2,pu}(x,u) \tag{8}$$

where,

$$w_1 + w_2 = 1 \tag{9}$$

The aforementioned MO-ORPD problem is mathematically a nonlinear constrained optimization problem. The decision variables including the control variables (i.e. $u$) and state variables (i.e. $x$) are as follows:

$$u^T = \left[\left[V_G\right]^T, \left[Q_C\right]^T, \left[T\right]^T\right]$$
$$x^T = \left[\left[V_L\right]^T, \left[Q_G\right]^T, \left[S_L\right]^T\right] \tag{10}$$

Since the objective functions Eqs. (1) and (2), do not have the same dimensions, in this paper, fuzzy satisfying method [31] is utilized to calculate the normalized (or per unit) form of both individual objective functions in Eq. (8). In the fuzzy satisfying method, a fuzzy membership number is defined for each objective function, which maps it to the interval [0, 1]. More generally, the *ith* objective function, $J_i$ is normalized as follows.

$$J_{i,pu} = \begin{cases} 1 & J_i \le J_i^{\min} \\ \dfrac{J_i - J_i^{\max}}{J_i^{\min} - J_i^{\max}} & J_i^{\min} \le J_i \le J_i^{\max} \\ 0 & J_i \ge J_i^{\max} \end{cases} \tag{11}$$

In this paper for objective functions Eqs. (1) and (2), the normalized values are expressed as:

$$PL_{pu} = J_{1,pu} = \frac{PL - PL^{\max}}{PL^{\min} - PL^{\max}} \tag{12}$$

$$VD_{pu} = J_{2,pu} = \frac{VD - VD^{\max}}{VD^{\min} - VD^{\max}} \tag{13}$$

After running the MO-ORPD for different values of weighting factors, to select the best compromising solution, fuzzy satisfying method based on logistic membership function is used. After normalization the objective functions best solution is selected by using min-max operator.

## 4. CASE STUDY

All coding is implemented in General Algebraic Modeling System (GAMS) environment and solved by SBB solver. Simulations are carried out on the IEEE 30-bus and IEEE 118-bus systems. The IEEE 30-bus system consists of 30 buses, which its 6 buses are generator bus. The network has 41 branches, 4 transformers and 9 capacitor banks [32]. Hence, according to Eq. (9), total number of control variables is 25.

The IEEE 118-bus system consists of 118 buses, with 54 generator buses. Bus 69 is the slack bus. The network has 186 branches, 9 transformers and 14 capacitor banks [32]. The total number of control variables is 78. The initial operating point of the systems are given in [33]. In order to clearly illustrate the effectiveness of proposed method, a comparison is made between the results of two different cases:

(A) Deterministic optimization (ignoring the uncertainty in load).

(B) Uncertainty characterization using Monte Carlo simulation.

The simulation results are described as follows.

### 4.1. Case I – IEEE 30-bus test system

### 4.1.1 Deterministic Optimization

In deterministic case, the actual value of load is considered in the multi objective optimal reactive power dispatch problem. Real power loss and voltage deviation are considered as conflicting objective functions through Eq. (8). In order to solve the MO-ORPD problem by weighted sum method, maximum and minimum values of the expected real power loss (i.e. $J_1$) and voltage deviation (i.e. $J_2$) are calculated, which are 1.6012MW, 1.2577MW, 0.034pu and 0.0011pu, respectively.

These border values are achieved by maximize-ing and minimizing $J_1$ and $J_2$ individually as the objective function of MO-ORPD. Table 1 shows the values of both objective functions for all 21 Pareto optimal solutions. As explained in Sec. 3, in order to select the best solution from the obtained Pareto optimal set, a fuzzy satisfying method is utilized here. It is evident from the last column of Table 1 that the best solution is *Solution#2*, with the maximum weakest membership number of 0.8291. The corresponding *PL* and *VD* are equal to 1.316 MW and 0.0056 pu, respectively. For the above Pareto optimal set, the Pareto optimal front is depicted in Fig. 1. In this figure, the optimal compromise solution (i.e. *Solution#2)* is also specified.

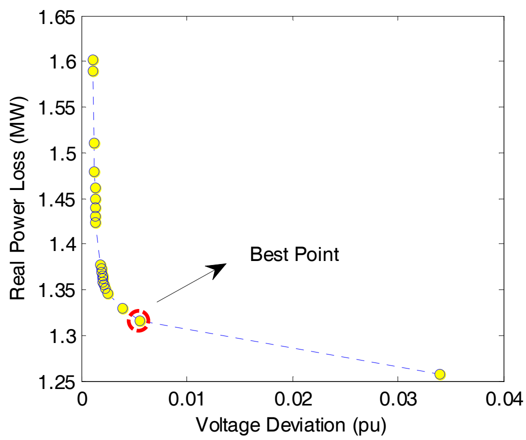


**Fig. 1**. Pareto optimal front for the IEEE 30-bus test system (Deterministic case)

**Table1**. Pareto optimal solutions for the IEEE 30-bus test system (Deterministic case)

| # | $W_1$ | $W_2$ | *PL or* ($J_1$) (MW) | *VD (or* $J_2$) (pu) | $J_{1,pu}$ | $J_{2,pu}$ | Min ($J_{1,pu}$, $J_{2,pu}$) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 1.2577 | 0.034 | 1 | 0 | 0 |
| 2 | 0.95 | 0.05 | 1.3164 | 0.0056 | 0.8291 | 0.8632 | 0.8291 |
| 3 | 0.9 | 0.1 | 1.329 | 0.0039 | 0.7924 | 0.9149 | 0.7924 |
| 4 | 0.85 | 0.15 | 1.3463 | 0.0025 | 0.7421 | 0.9574 | 0.7421 |
| 5 | 0.8 | 0.2 | 1.3511 | 0.0023 | 0.7281 | 0.9635 | 0.7281 |
| 6 | 0.75 | 0.25 | 1.3551 | 0.0022 | 0.7164 | 0.9666 | 0.7164 |
| 7 | 0.7 | 0.3 | 1.3587 | 0.0021 | 0.706 | 0.9696 | 0.706 |
| 8 | 0.65 | 0.35 | 1.362 | 0.002 | 0.6964 | 0.9726 | 0.6964 |
| 9 | 0.6 | 0.4 | 1.3655 | 0.002 | 0.6862 | 0.9726 | 0.6862 |
| 10 | 0.55 | 0.45 | 1.3692 | 0.0019 | 0.6754 | 0.9757 | 0.6754 |
| 11 | 0.5 | 0.5 | 1.3733 | 0.0019 | 0.6635 | 0.9757 | 0.6635 |
| 12 | 0.45 | 0.55 | 1.3778 | 0.0018 | 0.6504 | 0.9787 | 0.6504 |
| 13 | 0.4 | 0.6 | 1.423 | 0.0014 | 0.5188 | 0.9909 | 0.5188 |
| 14 | 0.35 | 0.65 | 1.4306 | 0.0014 | 0.4967 | 0.9909 | 0.4967 |
| 15 | 0.3 | 0.7 | 1.4393 | 0.0013 | 0.4713 | 0.9939 | 0.4713 |
| 16 | 0.25 | 0.75 | 1.4495 | 0.0013 | 0.4416 | 0.9939 | 0.4416 |
| 17 | 0.2 | 0.8 | 1.4622 | 0.0013 | 0.4047 | 0.9939 | 0.4047 |
| 18 | 0.15 | 0.85 | 1.4799 | 0.0012 | 0.3531 | 0.997 | 0.3531 |
| 19 | 0.1 | 0.9 | 1.5101 | 0.0012 | 0.2652 | 0.997 | 0.2652 |
| 20 | 0.05 | 0.95 | 1.5901 | 0.0011 | 0.0323 | 1 | 0.0323 |
| 21 | 0 | 1 | 1.6012 | 0.0011 | 0 | 1 | 0 |

### 4.1.2. Uncertainty modeling using MCS

In this section, a MCS-based procedure is considered to deal with the aforementioned load uncertainty [34]. The MCS is a numerical simulation procedure applied to the problems involving random variables with known or assumed probability distributions. It consists of repeating a deterministic simulation process, where in each simulation, a particular set of values for the random variables are generated according to their corresponding probability distributions. The results obtained in each iteration of MCS are similar to a deterministic simulation case. By collecting the results of many such MCS runs, it is possible to analyze the obtained results by statistical indices, such as mean (or average) value, standard deviation etc.

In the MCS the mean value ($\mu_{MCS}$) and standard deviation ($\sigma_{MCS}$) for a given variable (or parameter) $X$ are calculated as follows.

$$\mu_{MCS} = \frac{1}{N}\sum_{i=1}^{N} X_i \qquad (14)$$

$$\sigma_{MCS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \mu_{MCS})^2}$$

For load buses, the random variable to be considered in the MCS is load demand, due to its stochastic behavior. It is assumed that loads are normally distributed with a known mean value (corresponding to the forecasted value) and a known standard deviation in each bus. It is worth to note that, the mean value considered for each load, is its forecasted value which may be the peak or non-peak load. The appropriate values for each random variable are generally achieved from its probability distribution function or cumulative distribution function. In particular, the MATLAB function *randn* provides normally distributed random numbers directly. In this case, 10,000 random samples are picked up for considering the stochastic behavior of loads.

Here, for the sake of brevity just some statistical parameters such as mean, standard deviation and variance of the Pareto optimal solutions are reported. Table 2 gives the mean value of both objective functions for all 21 Pareto optimal solutions. Again, by using fuzzy satisfying method, *Solution#2* is the best. The Pareto optimal front of the objective functions is depicted in Fig. 2. The numerical values on the figure are mean, Standard Deviation (SD) and variance (Var) of the compromise optimal solution (i.e. *Solution#2*).

### 4.1.3. The obtained control variables

For *Solution#2* (i.e. the best compromise solution) the optimal values of control variables for both deterministic and probabilistic methods are given in Table 3. In the probabilistic case (i.e. by

MCS), the mean values for the control variables are given in this Table. One of the main advantages of MSC approach is that it gives the probability distribution of all uncertain variables. Histogram diagram is a proper tool for illustration of probability distribution. In this case, 4 randomly selected control variables and their corresponding probability distribution histograms are shown. The probability distributions for voltage of bus 11 ($V_{g11}$), real power output of generator located at bus 11 ($P_{g11}$), reactive power compensation in bus 24 and transformers tap changer between bus 28-27 are shown in Figs. 3-6, respectively.

**Table 2.** Pareto optimal solution for IEEE 30-bus test system (probabilistic case with MSC)

| # | $W_1$ | $W_2$ | PL (or $J_1$) MW) | VD (or $J_2$) (pu) | $J_{1,pu}$ | $J_{2,pu}$ | Min ($J_{1,pu}$, $J_{2,pu}$) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 1.262454 | 0.033635 | 1 | 0 | 0 |
| **2** | **0.95** | **0.05** | **1.321376** | **0.005165** | **0.8291** | **0.8728** | **0.8291** |
| 3 | 0.9 | 0.1 | 1.334024 | 0.003597 | 0.7924 | 0.9208 | 0.7924 |
| 4 | 0.85 | 0.15 | 1.351389 | 0.002306 | 0.7421 | 0.9604 | 0.7421 |
| 5 | 0.8 | 0.2 | 1.356207 | 0.002121 | 0.7281 | 0.9661 | 0.7281 |
| 6 | 0.75 | 0.25 | 1.360222 | 0.002029 | 0.7164 | 0.9689 | 0.7164 |
| 7 | 0.7 | 0.3 | 1.363836 | 0.001937 | 0.706 | 0.9717 | 0.706 |
| 8 | 0.65 | 0.35 | 1.367148 | 0.001845 | 0.6964 | 0.9746 | 0.6964 |
| 9 | 0.6 | 0.4 | 1.370662 | 0.001845 | 0.6862 | 0.9746 | 0.6862 |
| 10 | 0.55 | 0.45 | 1.374376 | 0.001752 | 0.6754 | 0.9774 | 0.6754 |
| 11 | 0.5 | 0.5 | 1.378491 | 0.001752 | 0.6635 | 0.9774 | 0.6635 |
| 12 | 0.45 | 0.55 | 1.383008 | 0.00166 | 0.6504 | 0.9802 | 0.6504 |
| 13 | 0.4 | 0.6 | 1.428379 | 0.001291 | 0.5188 | 0.9915 | 0.5188 |
| 14 | 0.35 | 0.65 | 1.436008 | 0.001291 | 0.4967 | 0.9915 | 0.4967 |
| 15 | 0.3 | 0.7 | 1.444741 | 0.001199 | 0.4713 | 0.9944 | 0.4713 |
| 16 | 0.25 | 0.75 | 1.454979 | 0.001199 | 0.4416 | 0.9944 | 0.4416 |
| 17 | 0.2 | 0.8 | 1.467727 | 0.001199 | 0.4047 | 0.9944 | 0.4047 |
| 18 | 0.15 | 0.85 | 1.485494 | 0.001107 | 0.3531 | 0.9972 | 0.3531 |
| 19 | 0.1 | 0.9 | 1.515808 | 0.001107 | 0.2652 | 0.9972 | 0.2652 |
| 20 | 0.05 | 0.95 | 1.596111 | 0.001015 | 0.0323 | 1 | 0.0323 |
| 21 | 0 | 1 | 1.607253 | 0.001015 | 0 | 1 | 0 |

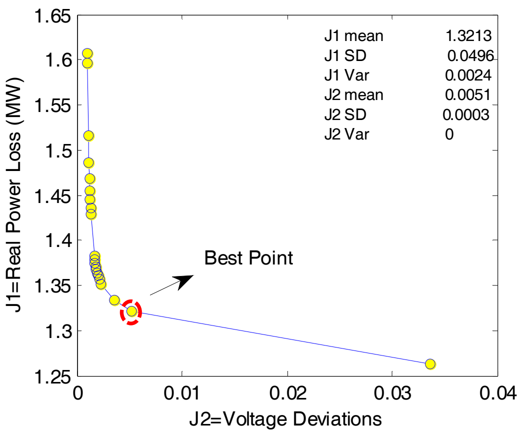


**Fig. 2**. Pareto optimal front for IEEE 30-bus test system (probabilistic case with MSC)

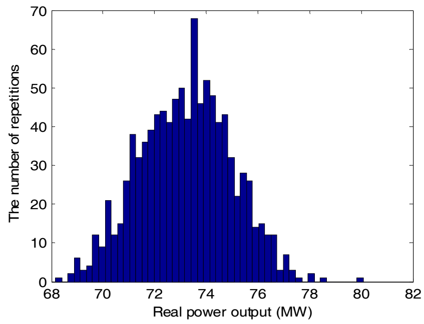


**Fig. 4.** The histogram of real power output of generator located at bus 11

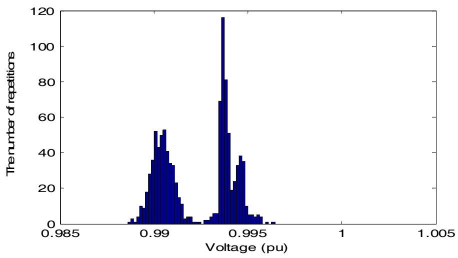


**Fig. 3.** The histogram of bus 11 voltage magnitude (pu)

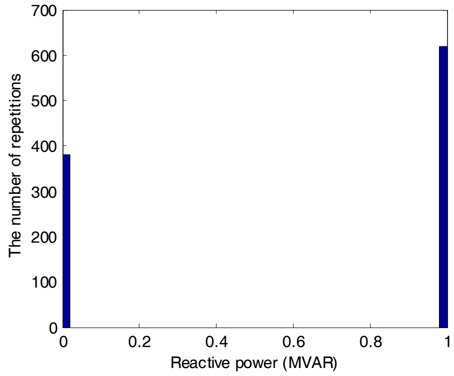


**Fig. 5**. The histogram of reactive power compensation at bus 24

**Table 3.** The obtained control variables for best compromise deterministic and probabilistic solutions (IEEE 30-bus system)

| Control Variable | Deterministic | probabilistic |
|---|---|---|
| $V_{g1}$(pu) | 0.998 | 1.0002 |
| $V_{g2}$(pu) | 0.998 | 1.0003 |
| $V_{g5}$(pu) | 0.9969 | 0.9989 |
| $V_{g8}$(pu) | 0.9986 | 1.0009 |
| $V_{g11}$(pu) | 0.9904 | 0.9923 |
| $V_{g13}$(pu) | 1.0098 | 1.01 |
| $P_{g1}$(MW) | 3.507 | 3.5177 |
| $P_{g2}$(MW) | 29.2979 | 29.8066 |
| $P_{g5}$(MW) | 100 | 99.5501 |
| $P_{g8}$(MW) | 45.3168 | 45.6626 |
| $P_{g11}$(MW) | 73.4394 | 73.2205 |
| $P_{g13}$(MW) | 33.1679 | 33.1967 |
| $Q_{c10}$(MVar) | 0 | 0 |
| $Q_{c12}$(MVar) | 0 | 0 |
| $Q_{c15}$(MVar) | 0 | 0 |
| $Q_{c17}$(MVar) | 10 | 8.05 |
| $Q_{c20}$(MVar) | 0 | 0.42 |
| $Q_{c21}$(MVar) | 10 | 10 |
| $Q_{c23}$(MVar) | 0 | 0 |
| $Q_{c24}$(MVar) | 10 | 6.2 |
| $Q_{c29}$(MVar) | 0 | 0 |
| $T_{6\text{-}9}$ | 0.9657 | 0.9672 |
| $T_{6\text{-}10}$ | 1.1 | 1.0895 |
| $T_{4\text{-}12}$ | 0.9906 | 0.9898 |
| $T_{28\text{-}27}$ | 0.9936 | 0.9926 |

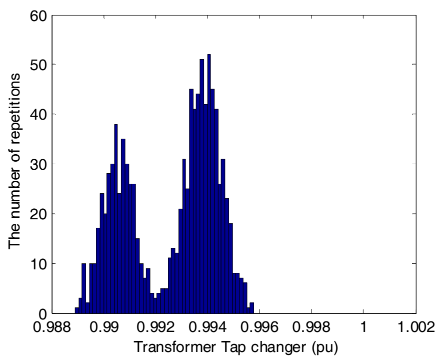


**Fig. 6.** . The histogram of tap ratio for the transformer between buses 27 and 28

## 4.2. Case II – IEEE118 bus test system

For the sake of brevity, in this case only the results obtained in probabilistic case are presented and for deterministic case only a summary of the obtained results are presented for the aim of comparison with MCS solutions.

### 4.2.1. Uncertainty modeling using MCS

In this case 10,000 different samples with normal PDF are selected. In this case, 11 different Pareto optimal solutions are derived. Table 4 summarizes the obtained results using MCS for case II. The corresponding Pareto front is depicted in Fig. 7.

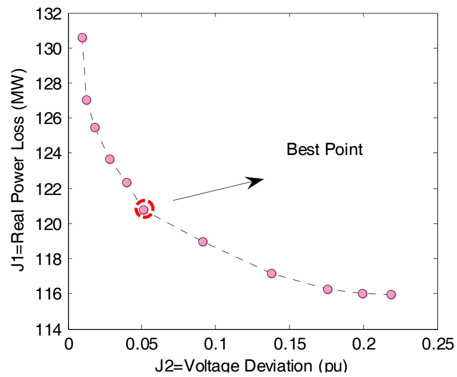


**Fig. 7**. Pareto optimal front for IEEE 118-bus test system

### 4.2.2. The obtained control variables

Table 5 summarizes the obtained control variables for the best compromise solutions in the deterministic and probabilistic approaches. It is noted worthy that in the probabilistic case the mean values for the control variables are given in this Table.

**Table 4**. Pareto optimal solution for IEEE 118-bus test system (probabilistic case with MSC)

| # | $W_1$ | $W_2$ | $PL$ ( $or\ J_1$) (MW) | $VD$ ($or\ J_2$) (pu) | $J_{1,pu}$ | $J_{2,pu}$ | Min ($J_{1,pu}$, $J_{2,pu}$) |
|---|---|---|---|---|---|---|---|
| 1 | 1.0 | 0.0 | 115.9607 | 0.2191 | 1 | 0 | 0 |
| 2 | 0.9 | 0.1 | 116.02456 | 0.1992 | 0.9956 | 0.0951 | 0.0951 |
| 3 | 0.8 | 0.2 | 116.2456 | 0.1761 | 0.9805 | 0.2054 | 0.2054 |
| 4 | 0.7 | 0.3 | 117.1425 | 0.1378 | 0.9192 | 0.3884 | 0.3884 |
| 5 | 0.6 | 0.4 | 118.9547 | 0.0914 | 0.7952 | 0.6101 | 0.6101 |
| **6** | **0.5** | **0.5** | **120.7648** | **0.0512** | **0.6714** | **0.8022** | **0.6714** |
| 7 | 0.4 | 0.6 | 122.3565 | 0.0395 | 0.5625 | 0.8581 | 0.5625 |
| 8 | 0.3 | 0.7 | 123.6466 | 0.0286 | 0.4742 | 0.9102 | 0.4742 |
| 9 | 0.2 | 0.8 | 125.4579 | 0.0184 | 0.3503 | 0.9589 | 0.3503 |
| 10 | 0.1 | 0.9 | 127.0545 | 0.0122 | 0.2411 | 0.9885 | 0.2411 |
| 11 | 0 | 1.0 | 130.5794 | 0.0098 | 0 | 1 | 0 |

**Table 5**. The obtained control variables for best compromise deterministic and probabilistic solutions (IEEE 118-bus system)

| Control Variable | Deterministic | probabilistic |
|---|---|---|
| Vg1 | 0.9902 | 1.028 |
| Vg4 | 1.0237 | 0.9985 |
| Vg6 | 1.0137 | 1.0145 |
| Vg8 | 1.0325 | 1.041 |
| Vg10 | 1.0459 | 1.0482 |
| Vg12 | 1.0109 | 1.0141 |
| Vg15 | 1.0072 | 1.0079 |
| Vg18 | 1.0091 | 1.0095 |
| Vg19 | 1.0058 | 1.0062 |
| Vg24 | 1.0202 | 1.0225 |
| Vg25 | 1.0473 | 1.0512 |
| Vg26 | 1.0547 | 1.0552 |
| Vg27 | 1.0116 | 1.0222 |
| Vg31 | 1.0037 | 1.0045 |
| Vg32 | 1.0084 | 1.0096 |
| Vg34 | 1.0166 | 1.0215 |
| Vg36 | 1.0138 | 1.0211 |
| Vg40 | 0.9989 | 1.0882 |
| Vg42 | 1.0032 | 1.0085 |
| Vg46 | 1.0145 | 1.0215 |
| Vg49 | 1.0316 | 1.0385 |
| Vg54 | 1.0063 | 1.0021 |
| Vg55 | 1.0048 | 1.0085 |
| Vg56 | 1.0051 | 1.0251 |
| Vg59 | 1.0253 | 1.0524 |
| Vg61 | 1.0256 | 1.0595 |
| Vg62 | 1.0225 | 1.0925 |
| Vg65 | 1.0344 | 1.0352 |
| Vg66 | 1.0443 | 1.0487 |
| Vg69 | 1.0503 | 1.0821 |
| Vg70 | 1.0158 | 1.0165 |
| Vg72 | 1.0111 | 1.0194 |
| Vg73 | 1.0111 | 1.0185 |
| Vg74 | 0.9949 | 0.9954 |
| Vg76 | 0.9802 | 0.9882 |
| Vg77 | 1.0212 | 1.0338 |
| Vg80 | 1.0325 | 1.0327 |
| Vg85 | 1.0315 | 1.0419 |
| Vg87 | 1.0105 | 1.0228 |
| Vg89 | 1.0574 | 1.0612 |
| Vg90 | 1.0293 | 1.0298 |
| Vg91 | 1.0282 | 1.0325 |
| Vg92 | 1.0352 | 1.0654 |
| Vg99 | 1.0215 | 1.0502 |
| Vg100 | 1.0298 | 1.0315 |
| Vg103 | 1.0191 | 1.0223 |
| Vg104 | 1.0089 | 1.0199 |
| Vg105 | 1.0063 | 1.0092 |
| Vg107 | 0.9944 | 0.9904 |
| Vg110 | 1.0021 | 1.0121 |
| Vg111 | 1.0094 | 1.0099 |
| Vg112 | 0.9879 | 0.9808 |
| Vg113 | 1.0166 | 1.0187 |
| Vg116 | 1.0262 | 1.0254 |
| Pg69(MW) | 500.7689 | 550.8577 |
| Qc5(MVar) | 0 | 0 |
| Qc34(MVar) | 2.8801 | |
| Qc37(MVar) | 0 | 0 |
| Qc44(MVar) | 10 | |
| Qc45(MVar) | 10 | |
| Qc46(MVar) | 10 | |
| Qc48(MVar) | 4.1331 | |
| Qc74(MVar) | 12 | |
| Qc79(MVar) | 20 | |
| Qc82(MVar) | 20 | |
| Qc83(MVar) | 10 | |
| Qc105(MVar) | 18.5654 | |
| Qc107(MVar) | 0 | 0 |
| Qc110(MVar) | 10 | |
| $T_{8-5}$ | 0.9982 | 0.9987 |
| $T_{26-25}$ | 1.056 | 1.058 |
| $T_{30-17}$ | 1.0025 | 1.0029 |
| $T_{38-37}$ | 0.9994 | 0.9995 |
| $T_{63-59}$ | 1.0022 | 1.0028 |
| $T_{64-61}$ | 1.0023 | 1.0151 |
| $T_{65-66}$ | 1.0023 | 1.0098 |
| $T_{68-69}$ | 1.0121 | 1.0138 |
| $T_{81-80}$ | 1.0102 | 1.0115 |

### 4.3. Discussion on the obtained results

Since the probabilistic MO-ORPD considering the load uncertainty is not reported in the previous literature, investigation of the performance of the proposed method is only possible by comparison of the obtained results in the deterministic case with the previously reported results in the literature.

Table 6 compares the obtained deterministic results for Cases I and II with the previously published works. In this table the results for minimization of both objective functions ($J_1$ and $J_2$) are compared with the heuristic methods.

In Tables 1, 2 and 4 the *solution*#1 is the case in which the only real power loss is minimized. *Solution* #11 is the case where the voltage deviations are minimized. It is observed from Table 6 that in both cases the probabilistic MCS-based approach results better solutions than the previously published methods.

**Table 6.** Comparison of obtained results for deterministic cases with previously published methods

| $J_1$ | Real Power Loss (MW) | | | |
|---|---|---|---|---|
| Method | Proposed | DE [13] | CPVEIHBMO [23] | QOTLBO [21] |
| Case I | 1.3164 | 4.8623 | 5.3243 | 5.2594 |
| Case II | 119.7686 | 129.579 | 124.0983 | 134.4059 |
| $J_2$ | Voltage Deviation (pu) | | | |
| Method | Proposed | DE [13] | CPVEIHBMO [23] | QOTLBO [21] |
| Case I | 0.0056 | 0.0911 | 0.87664 | 0.121 |
| Case II | 0.0498 | --- | 0.7397 | 0.24 |

## 5. CONCLUSIONS

This paper proposes a probabilistic approach for MO-ORPD problem. In this model, the technical constraints as well as the load uncertainty are taken into consideration. The stochastic nature of load is modeled using Monte Carlo simulations. Mixed integer nonlinear programming is used to solve the proposed probabilistic MO-ORPD problem. In order to evaluate the effectiveness of the proposed model, it is implemented on the IEEE 30-bus and IEEE-118 bus test systems. The numerical results demonstrate the effectiveness of the proposed methodology. The main advantages of this study are summarized as follow:

- Using MCS for load uncertainty modeling is a help system operator to have realistic decisions.
- The solutions obtained in the deterministic case, are better than the results attained by heuristic algorithms.
- The MCS approach gives the probability distribution of all output variables such as bus voltages, line flows etc. This is an important result, since the probability distribution of any uncertain parameter shows the statistical behavior of it. Hence, system operator can use the proposed MCS-based MO-ORPD problem for determination of optimal probability distribution of important variables such as power losses, voltage levels etc. “”

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